1. $\lim_{x\to 0} \cos\left(\frac{\pi - \pi\cos^2 x}{x^2}\right) =$

a) -1

b) 1

c) 0

d) 3π

e) $-\pi$

$$\lim_{x\to 0} \frac{\pi - \pi\cos^2 x}{x^2} = \lim_{x\to 0} \pi\left(\frac{\sin x}{x}\right)^2 = \pi$$

and since $\cos x$ is continuous at π, then

$$\lim_{x\to 0} \cos\left(\frac{\pi - \pi\cos^2 x}{x^2}\right) = \cos\left(\lim_{x\to 0} \frac{\pi-\pi\cos^2 x}{x^2}\right)$$

$$= \cos\pi = -1$$

2. If $xy + e^y = e$, then the value of y' at $x = 0$ is

a) $-\frac{1}{e}$

b) $-\frac{2}{e}$

c) $2e$

d) e

e) 0

$$x = 0 \Rightarrow e^y = e \Rightarrow y = 1$$

$$xy + e^y = e$$

$$\Rightarrow y + xy' + e^y y' = 0$$

$$\Rightarrow y + y'(x + e^y) = 0$$

$$\Rightarrow y' = \frac{-y}{x + e^y}$$

$$y'\Big|_{(0,1)} = \frac{-1}{0+e} = \frac{-1}{e}$$

3. If $y = 2^{x^2} + \ln(2x)$, then $\dfrac{dy}{dx} =$

a) $(\ln 4)\, x\, 2^{x^2} + \dfrac{1}{x}$

b) $(\ln 4)\, x\, 2^{x^2} + \dfrac{1}{2x}$

c) $x\, 2^{x^2} + \dfrac{1}{x}$

d) $x\, 2^{x^2} + \dfrac{1}{2x}$

e) $(\ln 2)\, x\, 2^{x^2} + \dfrac{1}{2x}$

$$\frac{dy}{dx} = 2x\, 2^{x^2} \ln 2 + \frac{2}{2x}$$

$$\Rightarrow \frac{dy}{dx} = (\ln 4)\, x\, 2^{x^2} + \frac{1}{x}$$

4. The sum of all value(s) of c satisfying the conclusion of the Mean Value Theorem for the function

$$f(x) = \frac{x^3}{3} - \frac{3}{2}x^2 + 2x + 1$$

on $[0, 3]$ is

a) 3

b) $\dfrac{3\sqrt{3}}{2}$

c) $\sqrt{3}$

d) 2

e) $2\sqrt{3}$

$$f(0) = 1, \quad f(3) = 9 - \frac{27}{2} + 7$$

$$= 16 - \frac{27}{2} = \frac{5}{2}$$

$$f'(x) = x^2 - 3x + 2$$

MVT, there is a number c in $(0,3)$ s.t

$$c^2 - 3c + 2 = \frac{\frac{5}{2} - 1}{3 - 0} = \frac{1}{2}$$

$$\Rightarrow 2c^2 - 6c + 4 = 1$$

$$\Rightarrow 2c^2 - 6c + 3 = 0$$

$$\therefore c = \frac{6 \pm \sqrt{36 - 24}}{4} = \frac{3 \pm \sqrt{3}}{2} \in (0,3)$$

$\therefore$ Sum $= 3$

5. If $f(x) = \dfrac{x^3 - 2x^2 + 5}{x^2 + 3x + 1}$, then an equation of the oblique (slant) asymptote for the graph of f is

a) $y = x - 5$
b) $y = x - 1$
c) $y = x + 2$
d) $y = x - 3$
e) $y = x + 6$

$$\begin{array}{r|l} & x - 5 \\ \hline x^2 + 3x + 1 & x^3 - 2x^2 + 5 \\ & -x^3 - 3x^2 - x \\ \hline & -5x^2 - x + 5 \\ & \pm 5x^2 \pm 15x \pm 5 \\ \hline & 14x + 10 \end{array}$$

$$f(x) = (x - 5) + \frac{14x + 10}{x^2 + 3x + 1}$$

6. If $f(x) = \begin{cases} a - x^2 & \text{if } x \le 1 \\ \ln x & \text{if } x > 1 \end{cases}$
is continuous on $(-\infty, \infty)$, then $a =$

a) 1
b) 2
c) -3
d) 0
e) -1

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$$

$$\Rightarrow \quad a - 1 = 0$$

$$\Rightarrow \quad \boxed{a = 1}$$

7. $\lim_{x \to 0} \frac{4^x - 2^x}{x} =$

$$\lim_{x \to 0} \frac{4^x - 2^x}{x} \quad \left(\frac{0}{0} \text{ Type}\right)$$

$$= \lim_{x \to 0} 4^x \ln 4 - 2^x \ln 2$$

$$= \ln 4 - \ln 2$$

$$= \ln 2$$

a) $\ln 2$
b) 0
c) 1
d) ∞
e) $-\infty$

8. Using Newton's Method to approximate one root of the equation $x^4 = x + 1$, we find that if $x_1 = 1$, then $x_2 =$

let $f(x) = x^4 - x - 1$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$f'(x) = 4x^3 - 1$$

$$x_2 = 1 - \frac{-1}{3} = 1 + \frac{1}{3} = \frac{4}{3}$$

a) $\frac{4}{3}$
b) $-\frac{1}{3}$
c) $\frac{2}{3}$
d) $\frac{-4}{3}$
e) 0

9. The linear approximation of $e^{\tan x}$ at $x = 0$ is

a) $1 + x$

b) $1 + ex$

c) $1 + \pi x$

d) $e + x$

e) $x - 1$

$f(x) = e^{\tan x} \Rightarrow f'(x) = e^{\tan x} \cdot \sec^2 x$

$f(x) \approx f(0) + f'(0)\,(x - 0)$

$e^{\tan x} \approx 1 + x$

10. The most antiderivative of $f(x) = (x+1)(2x-1)$ is

a) $\dfrac{2}{3}x^3 + \dfrac{1}{2}x^2 - x + c$

b) $2x^3 + x^2 - x + c$

c) $\left(\dfrac{x^2}{2} + x\right)(x^2 - x) + c$

d) $\dfrac{2}{3}x^3 - x^2 + c$

e) $\left(\dfrac{x^2}{2} + x + c_1\right)(x^2 - x + c_2)$

$f(x) = 2x^2 + x - 1$

$\therefore$

$F(x) = \frac{2}{3}x^3 + \frac{x^2}{2} - x + C$

11. If $f(x) = \dfrac{h(x)+x}{x+1}$, $f'(1) = \dfrac{1}{2}$, $h(1) = 1$ and $h(x)$ is differentiable, then $h'(1) =$

a) 1
b) 2
c) 0
d) $\dfrac{1}{2}$
e) $\dfrac{3}{2}$

$$f'(x) = \frac{(h'(x)+1)(x+1) - (h(x)+x)\cdot 1}{(x+1)^2}$$

$$\Rightarrow \frac{1}{2} = \frac{2(h'(1)+1) - 2}{4}$$

$$\Rightarrow h'(1)+1 = 2 \Rightarrow h'(1) = 1$$

12. If $f''(x) = e^x - 2\sin x$, $f(0) = 1$, $f(\pi) = \pi + e^\pi$, then $f\left(\dfrac{\pi}{2}\right) =$

a) $e^{\pi/2} + \dfrac{\pi}{2} + 2$
b) $e^{\pi/2} + \dfrac{\pi}{2}$
c) $e^{\pi/2} - \dfrac{\pi}{2}$
d) 0
e) $e^{\pi/2}$

$$f'(x) = e^x + 2\cos x + C_1$$

$$\Rightarrow f(x) = e^x + 2\sin x + C_1 x + C_2$$

$$f(0) = 1 \Rightarrow C_2 = 0,$$

$$f(\pi) = \pi + e^\pi \Rightarrow$$

$$e^\pi + C_1\pi = \pi + e^\pi$$

$$\Rightarrow C_1 = 1$$

$$\therefore f(x) = e^x + 2\sin x + x$$

$$f\left(\frac{\pi}{2}\right) = e^{\pi/2} + \frac{\pi}{2} + 2$$

13. The number of tangent lines to the curve $y = \tan x - \cot x$, $0 < x < 2\pi$ that are parallel to the line $y = 4x + 3$ is

$y' = \sec^2 x + \csc^2 x$, $y' = 4 \Rightarrow$

$\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} = 4 \Rightarrow \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} = 4$

$\Rightarrow (2 \sin x \cos x)^2 = 1 \Rightarrow \sin^2 2x = 1$

$\Rightarrow \sin 2x = 1 \Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}$

or $\sin 2x = -1 \Rightarrow x = \frac{3\pi}{4}, \frac{7\pi}{4}$

a) 4
b) 0
c) 1
d) 2
e) 3

14. Let f be the function $f(x) = \begin{cases} 1 - 2x^2 & \text{if } x \geq 1 \\ 5 - 4x & \text{if } x < 1 \end{cases}$ then $f'(1)$

a) does not exist
b) equals 4
c) equals -4
d) equals 1
e) equals -1

$\lim_{x \to 1^-} f(x) = 1$,

$\lim_{x \to 1^+} f(x) = -1$

$\therefore \lim_{x \to 1} f(x)$ DNE.

$\Rightarrow$ f is not continuous at $x = 1$

$\Rightarrow$ $f'(1)$ Does Not exist.

15. If $\left|\dfrac{9-4x^2}{3+2x}-6\right| < 0.1$ whenever $0 < \left|x+\frac{3}{2}\right| < \delta$ then the largest possible δ is

a) 0.05
b) 0.033
c) 0.025
d) 0.2
e) 0.4

$\left|\dfrac{(3-2x)(3+2x)}{3+2x}-6\right| < 0.1$ if $0 < \left|x+\frac{3}{2}\right| < \delta$

$|-3-2x| < 0.1$ if $0 < \left|x+\frac{3}{2}\right| < \delta$

$\Leftrightarrow 2\left|x+\frac{3}{2}\right| < 0.1$ if $0 < \left|x+\frac{3}{2}\right| < \delta$

$\Leftrightarrow \left|x+\frac{3}{2}\right| < 0.05$ if $0 < \left|x+\frac{3}{2}\right| < \delta$

$\therefore 0 < \delta \le 0.05$

16. The position of a particle is given by the equation

$$S = f(t) = \sin\left(\frac{\pi t}{2}\right), \qquad 0 \le t \le 4$$

when is the particle speeding up?

a) $1 < t < 2$ and $3 < t < 4$
b) $0 < t < 1$ and $3 < t < 4$
c) $0 < t < 2$ and $3 < t < 4$
d) $2 < t < 3$
e) $1 < t < 3$

$V(t) = S' = \frac{\pi}{2}\cos\left(\frac{\pi}{2}t\right)$

$a(t) = S'' = -\left(\frac{\pi}{2}\right)^2 \sin\left(\frac{\pi}{2}t\right)$

	(0,1)	(1,2)	(2,3)	(3,4)
$V(t)$	+	−	−	+
$a(t)$	−	−	+	+

$\therefore$ Speeding up on $(1,2)$ and $(3,4)$

17. The slope of the tangent line to the graph of $\tanh(x+y) + x\cosh y = 0$ at the point $(0,0)$ is equal to

a) -2
b) -1
c) 0
d) 1
e) 2

$$\operatorname{sech}^2(x+y)\cdot(1+y') + x\sinh y + \cosh y = 0$$

$$\Rightarrow \quad 1 + y'\big|_{(0,0)} + 1 = 0$$

$$\Rightarrow \quad y'\big|_{(0,0)} = -2$$

18. The absolute maximum of $f(x) = x + \dfrac{1}{x}$ in $[0.2, 4]$ is

a) 5.2
b) 7.25
c) 4.2
d) 2
e) 4.25

$$f'(x) = 1 - \frac{1}{x^2}$$

$$= \frac{(x-1)(x+1)}{x^2}$$

$0, 1 \notin$ Domain of $f(x)$

$\therefore$ f has only one critical point $x=1$

$$f(0.2) = 0.2 + \frac{1}{0.2} = 0.2 + 5 = 5.2$$

$$f(1) = 2$$

$$f(4) = 4 + 0.25 = 4.25$$

$\therefore$ Maximum $= 5.2$

19. If $f(1) = 10$ and $f'(x) \geq 2$ for $1 \leq x \leq 4$, then a possible choice for $f(4)$ is
(Hint: You may apply the Mean Value Theoreom)

a) 16
b) 14
c) 12
d) 10
e) 8

$$f'(c) = \frac{f(4) - f(1)}{3} \geq 2$$

$$f(4) \geq 6 + f(1)$$

$$f(4) \geq 16$$

20. The graph of $f(x) = 1 + \frac{1}{x} + \frac{1}{x^2}$

a) has one local minimum and is increasing on $(-2, 0)$
b) has two local maximum and is increasing on $(0, \infty)$
c) has no local maximum and is decreasing on $(-2, 0)$
d) has one local maximum and one local minimum
e) is increasing on $(-\infty, -2)$ and is decreasing on $(2, \infty)$.

$$f'(x) = -\frac{1}{x^2} - \frac{2}{x^3} = \frac{-x-2}{x^3}$$

Critical number $x = -2$

	$x < -2$	$-2 < x < 0$	$x > 0$
f'	$-$ ↘	$+$ ↗	$-$ ↘
x^3	$-$	$-$	$+$
$(-x-2)$	$+$	$-$	$-$

-2 0

$\therefore$ f has one Local minimum

and f is ↗ on $(-2, 0)$.

21. The sum of all critical points of the function $f(x) = \dfrac{x^2+1}{\sqrt{2x+1}}$ is

a) $\dfrac{1}{3}$

b) $-\dfrac{1}{2}$

c) $\dfrac{1}{6}$

d) $\dfrac{4}{3}$

e) $-\dfrac{3}{4}$

$$f'(x) = \frac{2x\sqrt{2x+1} - \dfrac{x^2+1}{\sqrt{2x+1}}}{2x+1}$$

$$= \frac{2x(2x+1) - x^2 - 1}{(2x+1)\sqrt{2x+1}} = \frac{3x^2+2x-1}{(2x+1)\sqrt{2x+1}}$$

$$= \frac{(3x-1)(x+1)}{(2x+1)\sqrt{2x+1}}$$

$\therefore\ x = \frac{1}{3}$ is the only critical number.

$f' = 0 \Rightarrow x = \frac{1}{3} \in D_f,\ x = -1 \notin D_f$

f' DNE $\Rightarrow x = -\frac{1}{2} \notin D_f$

22. If $f(x) = \dfrac{1+\tanh x}{1-\tanh x}$, then $f\left(\dfrac{1}{2}\right) =$

a) e

b) $\ln 2$

c) $2e$

d) $-\ln 2$

e) 2

$$f(x) = \frac{1 + \dfrac{e^x - e^{-x}}{e^x + e^{-x}}}{1 - \dfrac{e^x - e^{-x}}{e^x + e^{-x}}} = \frac{e^x + e^{-x} + e^x - e^{-x}}{e^x + e^{-x} - e^x + e^{-x}}$$

$$= \frac{2e^x}{2e^{-x}} = e^{2x}$$

$$\therefore f\left(\tfrac{1}{2}\right) = e$$

23. A rectangle has its base on the x-axis and its upper two vertices on the parabola $y = 12 - x^2$. What is the largest area the rectangle can have?

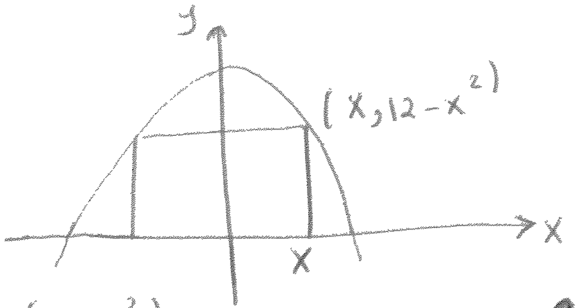
a) 32

b) 8

c) 64

d) 16

e) 24



$$A(x) = 2x(12 - x^2)$$
$$= 24x - 2x^3$$
$$A'(x) = 24 - 6x^2 = 0 \Rightarrow 6(2-x)(2+x) = 0$$
$$\Rightarrow x = \pm 2. \quad A_{max} = 32$$
$$A''(x) = -12x$$
$$A''(2) < 0$$

24. Let $f(x) = \dfrac{2e^x + 3e^{2x}}{e^{2x} - e^{3x}}$, then $f(x)$ has

a) one horizontal and one vertical asymptotes

b) one horizontal and two vertical asymptotes

c) two horizontal asymptotes

d) no vertical asymptotes

e) no horizontal aysmptotes

H.A: $\displaystyle \lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{2e^{-2x} + 3e^{-x}}{e^{-x} - 1} = 0$. $\therefore y = 0$ is H.A

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{2e^{-x} + 3}{1 - e^{x}} = \infty$$

V.A: $e^{2x} - e^{3x} = 0 \Rightarrow x = 0$

$$\lim_{x\to 0} |f(x)| = \lim_{x\to 0} \left| \frac{2e^{x} + 3e^{2x}}{e^{2x} - e^{3x}} \right| = \infty$$

$\therefore x = 0$ is V.A

25. Consider the function $f(x) = \dfrac{x-1}{x^2}$. Which of the following statements is true about the graph of f:

$f'(x) = \dfrac{x-1}{x^2}$, $f''(x) = \dfrac{2(x-3)}{x^4}$

f'' $\quad (-) \quad (-) \quad (+)$

$-\infty \quad 0 \quad 3 \quad \infty$

a) The graph has one inflection point only.
b) The graph has two inflection points.
c) The graph is concaving downward on $(3, \infty)$
d) The graph is concaving upward on $(-\infty, 0)$
e) The graph has no inflection points.

26. A tank in the shape of a right circular cylinder is being filled with water. The radius of the base is 3 meters. If the water is being pumped into the tank at a rate of $2\,m^3/min$, then the water level is rising at the rate of (in meter per minute).

a) $\dfrac{2}{9\pi}$
b) $\dfrac{1}{9\pi}$
c) $9\,\pi$
d) $\dfrac{9\pi}{2}$
e) $\dfrac{2}{3\pi}$

$V = \pi r^2 h = 9\pi h$

$\dfrac{dV}{dt} = 9\pi \dfrac{dh}{dt}$

$\Rightarrow 2 = 9\pi \dfrac{dh}{dt}$

$\therefore \dfrac{dh}{dt} = \dfrac{2}{9\pi}$ m/min.

27. $\lim\limits_{x\to 0+} [\cos(2x)]^{1/x^2} =$

1^{∞} Type

a) e^{-2}

b) e

c) ∞

d) 1

e) 0

$$y = (\cos(2x))^{\frac{1}{x^2}} \Rightarrow \ln y = \frac{\ln(\cos(2x))}{x^2}$$

$$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \frac{\ln(\cos(2x))}{x^2} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x\to 0^+} \frac{-2\frac{\sin(2x)}{\cos(2x)}}{2x}$$

$$= \lim_{x\to 0^+} \frac{\sin(2x)}{2x} \cdot \frac{-2}{\cos(2x)} = -2$$

$$\therefore \lim_{x\to 0^+} y = e^{-2}$$

28. A right triangle whose hypotenuse is $\sqrt{3}\,m$ long is revolved about one of its legs to generate a right circular cone. Find the volume of the cone of greatest volume that can be made this way.

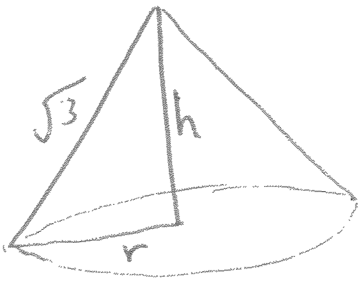
a) $\frac{2\pi}{3}\,m^3$

b) $3\pi\,m^3$

c) $\frac{9\pi}{2}\,m^3$

d) $4\pi\,m^3$

e) $\frac{7\pi}{3}\,m^3$



$$r^2 + h^2 = 3$$

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi(3h - h^3)$$

$$V'(h) = \frac{1}{3}\pi(3 - 3h^2) = 0 \Rightarrow h = 1$$

$$V_{max} = \frac{1}{3}\pi(3-1)\cdot 1$$

$$= \frac{2\pi}{3}$$