King Fahd University of Petroleum and Minerals
Math. & Stat. Department
161-Math 101 Quiz (2)

Name	ID	SEC 01

Q1) Use $\varepsilon-\delta$ definition of limit to show that $\lim\limits_{x\to 2}\dfrac{3x^2-4x-4}{x-2}=8$.

$$\frac{3x^2-4x-4}{x-2}=\frac{(3x+2)(x-2)}{(x-2)}=3x+2$$

① For all ε there exist a δ s.t. if $|x-2|<\delta \Rightarrow |3x+2-8|<\varepsilon$.

② $|3x+2-8|<\varepsilon$ $\quad -\varepsilon+6<3x<\varepsilon+6$, $\dfrac{-\varepsilon+6}{3}<x<\dfrac{\varepsilon+6}{3}$

$-\dfrac{\varepsilon}{3}+2<x<\dfrac{\varepsilon}{3}+2$, $-\dfrac{\varepsilon}{3}<x-2<\dfrac{\varepsilon}{3}$

③ $\delta=\varepsilon/3$

$|x-2|<\varepsilon/3 \Rightarrow |3x-6|<\varepsilon \Rightarrow |3x+2-8|<\varepsilon$.

Q2) Find all horizontal assymptotes of the function $f(x)=\dfrac{5-4x^3}{\sqrt{x^6-x^4}}$.

$$\lim_{x\to\infty}\frac{5-4x^3}{\sqrt{x^6}\sqrt{1-x^4/x^6}}=\lim_{x\to\infty}\frac{5-4x^3}{|x^3|\sqrt{1-1/x^2}}$$

$$=\lim_{x\to\infty}\frac{5-4x^3}{x^3\sqrt{1-1/x^2}}=\lim_{x\to\infty}\frac{\overset{0}{5/x^3}-4}{\sqrt{1-\underset{0}{1/x^2}}}=-4$$

$$\lim_{x\to-\infty}\frac{5-4x^3}{|x^3|\sqrt{1-1/x^2}}=\lim_{x\to-\infty}\frac{5-4x^3}{-x^3\sqrt{1-1/x^2}}$$

$$=\lim_{x\to-\infty}\frac{5/x^3-4}{-\sqrt{1-1/x^2}}=4$$

$y=4$, $y=-4$ are horizontal assymptotes.

Q3) Use intermediate value theorem(IVT) to show that $\cos x=x^2$ has at least two solutions in the interval $(-\pi/2,\pi/2)$.

$f(x)=\cos x-x^2$ is cont. function.

$f(-\pi/2)=\cos(-\pi/2)-(-\pi/2)^2=-\pi^2/4<0$

$f(0)=\cos(0)-0^2=1>0$

$f(\pi/2)=\cos\pi/2-(\pi/2)^2=-\pi^2/4<0$.

By IVT in the intervals $[-\pi/2,0]$ and $[0,\pi/2]$ there are $c_1\in[-\pi/2,0]$ and $c_2\in[0,\pi/2]$ such that

$f(c_1)=0 \qquad f(c_2)=0$.