(1) Sketch the graph of a function $f$ that satisfies all of the given conditions:

\[
\begin{align*}
\lim_{x \to -4^-} f(x) &= -\infty; \quad \lim_{x \to -4^+} f(x) = \infty; \\
\lim_{x \to -\infty} f(x) &= 0; \quad \lim_{x \to -2} f(x) = 1; \\
f \text{ is undefined at } -2; \quad \lim_{x \to 1^-} f(x) &= -1; \\
\lim_{x \to 1^+} f(x) &= 2; \quad f(1) = 1
\end{align*}
\]
(2) Evaluate the limit, if it exists:

(a) \( \lim_{x \to 3} \frac{|x^2 - 9|}{x - 3} \)

(b) \( \lim_{x \to 1} \frac{4 - x^2}{2 - x - x^2} \)

(c) \( \lim_{x \to -2} \left[ \left[ \frac{1}{2} x + 1 \right] \right] \), where \([.]\) denotes the greatest integer function.
(d) \[ \lim_{x \to +\infty} \frac{2x + x \cos x}{5x^2 - 2x + 1}. \]

(e) \[ \lim_{x \to 0^+} x \sin \left( \frac{\sqrt{x^2 + 2}}{x} \right). \]

(e) \[ \lim_{x \to -\infty} (-33x + 1)^3 (2x - 1)^2 x \]
(3) Find the horizontal asymptotes of the graph of the function \( f(x) = \tan^{-1} \left( \frac{\sqrt{9x^2+2}}{3x+7} \right) \).

(4) Use the graph of \( f(x) = 2\sqrt{x} \) to find a number \( \delta \) such that \( |2\sqrt{x} - 4| < 1 \) whenever \( |x - 4| < \delta \). (Show your work and write your answer in simplest rational form \( \frac{p}{q} \).)
(5) Use the Intermediate Value Theorem to show that there is a root of the equation $e^{-x^2} = x$ between 0 and 1.
(6) Find the values of $a$ and $b$ that make the function

\[ f(x) = \begin{cases} 
 3 & \text{if } x = 1 \\
 ax^2 - bx + 3 & \text{if } 1 < x < 2 \\
 2x - a + b & \text{if } 2 \leq x < 3 \\
 6 & \text{if } x = 3.
\end{cases} \]

continuous on the closed interval $[1, 3]$. (Use limits to justify your steps)

(7) Given the function $f(x) = \frac{2x^2 + kx - 14}{x-2}$, where $k$ is a constant, find $k$ such that $x = 2$ is a removable discontinuity of $f$. (Give reasons to your steps).
(8) The displacement (in meters) of a particle moving in a straight line is given by \( s = \frac{1}{\sqrt{5-t}} \) where \( t \) is measured in seconds. Use limits to find the instantaneous velocity of the particle when \( t = 1 \).
(9) Prove that \( \lim_{x \to 1} 2x + 2 = 4. \)
(10) Find the equation of the tangent line to $f(x) = x - \frac{1}{x}$ at $x = 3$. 
(11) Show that \( f(x) = \sqrt{16 - x} \) is continuous on the interval \((-\infty, 16]\).

(12) Let

\[
  f(x) = \begin{cases} 
    x^2 & \text{if } x \leq 2 \\
    mx + r & \text{if } x > 2.
  \end{cases}
\]

Find the values of \( m \) and \( r \) that make \( f \) differentiable everywhere.