

MATH 102.1 (Term 161)

Quiz 1 (Sects. 5.3, 5.4 & 5.5)

Duration: 20min

Name: _____

ID number: _____

1.) (4pts) Evaluate $\lim_{n \rightarrow \infty} \sum_{i=1}^n x_i^2 (x_i^3 + 1)^6 \Delta x$ by expressing this limit as a definite integral on the interval $[0, 1]$.

2.) (6pts) Evaluate $A = \int_2^3 (x-2)^8 (x+3) dx$, $B = \int_{-1}^0 x^9 \sqrt[4]{x^5 + 1} dx$, $C = \int_{\pi/4}^{\pi/3} \frac{\sin 2x}{(1 + \cos^2 x)^5} dx$

$$1.) \lim_{n \rightarrow \infty} \sum_{i=1}^n x_i^2 (x_i^3 + 1)^6 \Delta x = \int_0^1 x^2 (x^3 + 1)^6 dx$$

$$u = x^3 + 1, \quad du = 3x^2 dx$$

$$\int_0^1 x^2 (x^3 + 1)^6 dx = \frac{1}{3} \int_1^2 u^6 du = \frac{1}{21} [u^7]_1^2$$

$$= \frac{127}{21}$$

$$2.) A = \int_2^3 (x-2)^8 (x+3) dx$$

$$u = x-2, \quad du = dx$$

$$A = \int_0^1 u^8 (u+5) du = \int_0^1 (u^9 + 5u^8) du$$

$$= \left[\frac{u^{10}}{10} + \frac{5u^9}{9} \right]_0^1 = \frac{1}{10} + \frac{5}{9} = \frac{59}{90}$$

$$B = \int_{-1}^0 x^9 \sqrt[4]{x^5 + 1} dx$$

$$u = x^5 + 1, \quad du = 5x^4 dx$$

$$B = \frac{1}{5} \int_0^1 u^{1/4} (u-1) du$$

$$= \frac{1}{5} \int_0^1 (u^{5/4} - u^{1/4}) du$$

$$= \frac{1}{5} \left[\frac{4}{9} u^{9/4} - \frac{4}{5} u^{5/4} \right]_0^1$$

$$= \frac{1}{5} \left(\frac{4}{9} - \frac{4}{5} \right) = -\frac{16}{225}$$

$$C = \int_{\pi/4}^{\pi/3} \frac{\sin 2x}{(1 + \cos^2 x)^5} dx$$

$$u = 1 + \cos^2 x, \quad du = -2 \cos x \sin x dx = -\sin 2x dx$$

$$C = - \int_{3/2}^{5/4} \frac{du}{u^5} = - \int_{3/2}^{5/4} u^{-5} du$$

$$= \left[\frac{u^{-4}}{-4} \right]_{3/2}^{5/4}$$

$$= \frac{1}{4} \left(\left(\frac{5}{4} \right)^{-4} - \left(\frac{3}{2} \right)^{-4} \right)$$