

MATH 102.1 (Term 161)

Quiz 3 (Sects. 7.1, 7.2 & 7.3)

Duration: 20min

Name: _____

ID number: _____

- 1.) (3pts) Evaluate the integral $I = \int \cos^4 x dx$.
 2.) (4pts) Evaluate the integral $J = \int \frac{1}{x^2 \sqrt{4x^2 - 25}} dx$.
 3.) (3pts) Evaluate the integral $K = \int (1+x^2) \sin x dx$.

1*) $\cos^2 x = \frac{1 + \cos 2x}{2}$

$$I = \int \left(\frac{1 + \cos 2x}{2} \right)^2 dx$$

$$= \frac{1}{4} \int (1 + 2\cos 2x + \cos^2 2x) dx$$

$$= \frac{1}{4} \int \left(1 + 2\cos 2x + \frac{1 + \cos 4x}{2} \right) dx$$

$$= \frac{1}{4} \int \left(\frac{3}{2} + 2\cos 2x + \frac{\cos 4x}{2} \right) dx$$

$$= \frac{3}{8}x + \frac{\sin 2x}{4} + \frac{\sin 4x}{32} + C$$

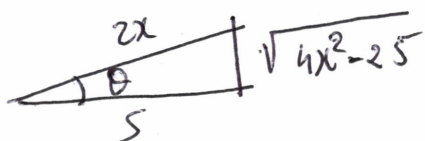
2*) $J = \int \frac{dx}{x^2 \sqrt{4x^2 - 25}}$

$2x = 5 \sec \theta, \quad 0 \leq \theta < \pi/2$
 $2dx = 5 \sec \theta \tan \theta d\theta$

$$J = \int \frac{5/2 \sec \theta \tan \theta d\theta}{\frac{25}{4} \sec^2 \theta \cdot 5 \tan \theta}$$

$$= \frac{2}{25} \int \cos \theta d\theta$$

$$= \frac{2}{25} \sin \theta + C$$



$$J = \frac{2}{25} \frac{\sqrt{4x^2 - 25}}{2x} + C$$

3*) Integration by parts.

$u = 1+x^2 \rightarrow u' = 2x$
 $v' = \sin x \rightarrow v = -\cos x$

$$K = -(1+x^2) \cos x + \int 2x \cos x dx$$

$u = 2x \rightarrow u' = 2$
 $v' = \cos x \rightarrow v = \sin x$

$$K = -(1+x^2) \cos x + 2x \sin x - 2 \int \sin x dx$$

$$= -(1+x^2) \cos x + 2x \sin x + 2 \cos x + C$$