MATH 102.1 (Term 161)

Quiz 4 (Sects. 7.4, 7.8 & 8.1) Duration: 20min

Name: ID number:

1.)(pts)Evaluate the integral $I = \int \frac{1}{x(x^2-2x-3)}dx$.

2.)(pts)Does the integral $J = \int_0^1 \frac{\ln x}{x}dx$ converge or diverge?

3.)(3pts)Find the arc length of the curve $y = \frac{x^4}{4} + \frac{1}{8x^2}$ for $1 \le x \le 2$.

1.) $\frac{1}{x(x^2-2x-3)} = \frac{a}{x} + \frac{b}{x+1} + \frac{c}{x-3}$

$a = \frac{1}{x^2-2x-3}\Big|_{x=0} = -\frac{1}{3}$

$b = \frac{1}{x(x-3)}\Big|_{x=-1} = 1/4$

$c = \frac{1}{x(x+1)}\Big|_{x=3} = \frac{1}{12}$

$I = \int \left(\frac{-\frac{1}{3}}{x} + \frac{\frac{1}{4}}{x+1} + \frac{\frac{1}{12}}{x-3} \right) dx$

$= -\frac{1}{3}\ln|x| + \frac{1}{4}\ln|x+1| + \frac{1}{12}\ln|x-3| + C$

2.) $\int_t^1 \frac{\ln x}{x}\,dx = \left[\frac{(\ln x)^2}{2} \right]_t^1$

$= -\frac{(\ln t)^2}{2}$

$\lim_{t \to 0^+} \left(-\frac{(\ln t)^2}{2} \right) = -\infty$

Thus, J diverges.

3.) $f(x) = \frac{x^4}{4} + \frac{1}{8x^2}$

$f'(x) = x^3 - \frac{1}{4x^3}$

$1 + f'(x)^2 = 1 + \left(x^3 - \frac{1}{4x^3}\right)^2$

$= 1 + \left(x^6 - \frac{1}{2} + \frac{1}{16x^6}\right)$

$= x^6 + \frac{1}{2} + \frac{1}{16x^6}$

$= \left(x^3 + \frac{1}{4x^3}\right)^2$

$L = \int_1^2 \left(x^3 + \frac{1}{4x^3}\right) dx$

$= \left[\frac{x^4}{4} - \frac{1}{8}\frac{1}{x^2} \right]_1^2$

$= 4 - \frac{1}{32} - \frac{1}{4} + \frac{1}{8}$

$= \frac{123}{32}$.