MATH 102.1 (Term 161)

Quiz 6 (Sects. 11.4, 11.5 & 11.6) Duration: 20min

Name: ID number:

1.)(4pts)Study the convergence of the series $\sum_{n=0}^{\infty} \frac{\sin(n^2)}{(n+1)^{3/2}}$ and $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$.

2.)(4pts)Study the convergence of the series $\sum_{n=1}^{\infty} \frac{(n)!}{(2n)^n}$ and $\sum_{n=1}^{\infty} \frac{n^{11}+2}{n^2(n^5+3)^2}$.

3.)(4pts)Study the convergence of the series $\sum_{n=1}^{\infty} \left(\frac{1+2\ln\sqrt{n}}{1+\sqrt{n}}\right)^{-n}$ and $\sum_{n=1}^{\infty} \left(\frac{1+2n^2}{n^2+1}\right)^{-2n}$.

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1) a) $\frac{|\sin n^2|}{(n+1)^{3/2}} \le \frac{1}{(n+1)^{3/2}}$

$\sum \frac{1}{n}$ conv $\Rightarrow \sum \frac{|\sin n^2|}{(n+1)^{3/2}}$ conv

Thus, $\sum_{n\ge 0}^{\infty} \frac{\sin n^2}{(n+1)^{3/2}}$ absolutely converges

b) $\cos n\pi = (-1)^n$

$\sum_{n\ge 1}^{\infty} \frac{\cos n\pi}{n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

Alternating harmonic series

$\Rightarrow \sum a_n$ converges

2.) $\frac{a_{n+1}}{a_n} = \frac{(n+1)!}{(2n+2)^{n+1}} \frac{(2n)^n}{n!}$

$= \frac{n+1}{2n+2} \left(\frac{2n}{2n+2}\right)^n$

$\left(\frac{2n}{2n+2}\right)^n = \left(\frac{n}{n+1}\right)^n = e^{n\ln\left(\frac{n}{n+1}\right)}$

$n\ln\left(\frac{n}{n+1}\right) = \frac{\ln\left(\frac{n}{n+1}\right)}{\frac{1}{n}} \xrightarrow{HR} \frac{\frac{1}{n(n+1)}}{-\frac{1}{n^2}}$

$\longrightarrow -1$

So, $\frac{n+1}{2n+2} \to \frac{1}{2}$

$\left(\frac{2n}{2n+2}\right)^n \longrightarrow e^{-1}$

$\Rightarrow \frac{a_{n+1}}{a_n} \longrightarrow \frac{1}{2e} < 1$

$\Rightarrow \sum a_n$ converges

b) $a_n = \frac{n^{11}+2}{n^2(n^5+3)^2}$, $b_n = \frac{1}{n}$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n^{11}+2}{n(n^5+3)^2} = 1$

$\sum \frac{1}{n}$ diverges $\Rightarrow \sum a_n$ diverges

3.) a) $\sqrt[n]{a_n} = \left(\frac{1+2\ln\sqrt{n}}{1+\sqrt{n}}\right)^{-1} = e^{-\ln\left(\frac{1+2\ln\sqrt{n}}{1+\sqrt{n}}\right)}$

$\lim_{n\to\infty} \sqrt[n]{a_n} = +\infty > 1$

$\Rightarrow \sum a_n$ diverges

b) $\sqrt[n]{a_n} = \left(\frac{1+2n^2}{n^2+1}\right)^{-2} = e^{-2\ln\left(\frac{1+2n^2}{n^2+1}\right)}$

$\lim_{n\to\infty} \sqrt[n]{a_n} = e^{-2\ln 2} = 2^{-2} = \frac{1}{4} < 1$

$\Rightarrow \sum a_n$ converges.