Exercise 1 (12 pts).

(1) Explain why $9 \equiv 2 \pmod{7}$ and $4 \equiv -3 \pmod{7}$.

(2) Show that for each integer $n \geq 0$, $3^{2n+1} + 2^{n+2}$ is divisible by 7.

(3) If today is Saturday, then what day will be after $(3^{2017} + 2^{1010} + 2)$ days?
Exercise 2 (10 pts). Let $x \in \mathbb{R}$. Show that:

1. If $\left| \frac{\sin x}{x^2 - x + 1} \right| > 2$, then $e^x > 2016$.

2. If $|x - 1| < 2017$, then $\frac{e^{-x^2}}{x^2 - 2x + 5/4} \leq 4$. 
Exercise 3 (12 pts). Let $a_1, a_2, b_1, b_2 \in \mathbb{R}$.

(i) Show that if $a_1 \geq a_2$ and $b_1 \geq b_2$, then

$$a_1 b_1 + a_2 b_2 \geq a_1 b_2 + a_2 b_1.$$ 

(ii) Show that for all real numbers $\alpha, \beta$, we have

$$\alpha^2 + \beta^2 \geq 2\alpha\beta.$$ 

(iii) Use the result of (i) to show that for all real numbers $\alpha, \beta$, we have

$$\alpha^2 + \beta^2 \geq 2\alpha\beta.$$
Exercise 4 (8 pts). Show that if $n$ is not divisible by 3, then $n^2 + 2$ is divisible by 3.
Exercise 5 (12 pts). Let $a \in \mathbb{Z}$. Show that if $a$ is a perfect square (i.e., there is $a \in \mathbb{Z}$, such that $a = b^2$), then $a \not\equiv 2 \pmod{4}$. 
Exercise 6 (8 pts). Let $P_1, P_2, \ldots, P_n$ and $Q$ be statements. Show that:

$$[(P_1 \lor P_2, \ldots \lor P_n) \rightarrow Q] \equiv [(P_1 \rightarrow Q) \land (P_2 \rightarrow Q) \land \ldots \land (P_n \rightarrow Q)].$$
Exercise 7 (38 pts). For $P, Q$ statements, we denote by $P \oplus Q$ the statement $(P \lor Q) \land \overline{P} \land \overline{Q}$. Show that the following properties hold:

1. $P \oplus Q \equiv (P \land \overline{Q}) \lor (Q \land \overline{P})$.
2. $P \oplus \overline{Q} \equiv (P \iff Q)$.
3. $P \oplus Q \equiv \overline{P} \oplus \overline{Q}$.
4. If $C$ is a contradiction, then $P \oplus C \equiv P$ and $P \oplus P \equiv C$.
5. $(P \oplus Q) \oplus R \equiv P \oplus (Q \oplus R)$.
6. $(P \oplus Q) \land R \equiv (P \land R) \oplus (Q \land R)$. 
Use the results of the previous questions (5) and (6) to show that, if $A, B$ are subsets of a universal set $U$, then we have:

(7) $(A \Delta B) \Delta C = A \Delta (B \Delta C)$.  
(8) $(A \Delta B) \cap C = (A \cap C) \Delta (B \cap C)$. 