Exercise 1 (10 pts). Let $\phi$ be the positive root of the quadratic equation

$$x^2 - x - 1 = 0.$$ 

Let $(F_n, n \in \mathbb{N})$ be the sequence defined recursively by $F_0 = 1, F_1 = 1,$ and $F_n = F_{n-1} + F_{n-2},$ for $n > 1.$

Show that $F_n \leq \phi^n,$ for each integer $n \geq 0.$
Exercise 2 (10 pts). Let \((u_n, n \in \mathbb{N})\) be the sequence defined recursively by \(u_0 = 0, u_1 = 1, u_{n+1} = u_n + 2u_{n-1}\), for \(n \geq 0\).

1. Show that \(u_n \in \mathbb{N}\), for each \(n \in \mathbb{N}\).

2. Show that \(u_n = \frac{1}{3}(2^n - (-1)^n)\), for each \(n \in \mathbb{N}\).
Exercise 3 (10 pts). Let \((u_n, n \in \mathbb{N})\) be the sequence defined by \(u_0 = 1\), \(u_1 = 2\) and \(u_{n+2} = 5u_{n+1} - 6u_n\), for \(n \geq 0\).

(1) Find \(u_2, u_3, u_4\).

(2) Find a formula for \(u_n\).
Exercise 4 (10 pts).

1. Show that the interval \([0, 1]\) equipped with the usual order is not well ordered.

2. Show that the set \(\{2^n \mid n \in \mathbb{N}\}\) equipped with the usual order is well ordered.
Exercise 5 (15 pts). Let \( \sim \) be the relation defined on \( \mathbb{Z} \) by;
\[
x \sim y \iff x^2 \equiv y^2 \pmod{5}
\]

(1) Show that \( \sim \) is an equivalence relation.
(2) Find \( 0, 1, 2 \).
(3) Find the cardinality of the quotient set \( \mathbb{Z}/\sim \).
Exercise 6 (10 pts). Let $R$ be the binary relation defined on $\mathbb{R}^2$ by:

$$(x, y) R (x', y') \text{ if and only if } |x' - x| \leq |y' - y|.$$ 

Show that $R$ is an ordering on $\mathbb{R}^2$ (reflexive, antisymmetric and transitive).
Exercise 7 (15 pts).

(1) Give the table of multiplication on \( \mathbb{Z}_8 \).

(2) Deduce from (1) the solution of the congruence equation:

\[ 2x \equiv 4 \pmod{8}, \]

where the unknown \( x \) is in \( \mathbb{Z} \).

(3) Deduce from (1) the solution of the congruence equation:

\[ x^2 \equiv 4 \pmod{8}, \]

where the unknown \( x \) is in \( \mathbb{Z} \).