King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics

MATH 302, Semester 161 (2016-2017)

EXAM I

October 26, 2016

Allowed Time: 2 Hours

Student Name:

Student ID Number:

Section Number:

Serial Number:

Instructor's Name:

Instructions:

1. Write neatly and legibly -- *you may lose points for messy work*.
2. Show all your work -- *no points for answers without justification*.
3. Programmable Calculators and Mobiles are <u>not</u> allowed.
4. Make sure that you have 6 different problems (6 pages + cover page).

Problem No.	Points	Maximum Points
1		15
2		15
3		20
4		20
5		15
6		10
Total:		100

Coordinator: Dr Ali N Duman

Q1. (a) Determine whether, $S = \{\langle x, y, z, t\rangle \mid xy = zt\}$ is a subspace of $\mathbb{R}^4$? ***[5 points]***

(b) Show that $V = \{\langle x, y, z, w\rangle \mid x + 2y = z + 3w = 0\}$ is a subspace of $\mathbb{R}^4$. ***[5 points]***

(c) Find a basis and for the subspace, $V = \{\langle x, y, z, w\rangle \mid x + 2y = z + 3w = 0\}$ of $\mathbb{R}^4$. What is the dimension of this subspace? ***[5 points]***

Q2. Using **Gaussian elimination**, find the currents in all branches of the circuit below.

[15 points]

Hint: First, reduce the number of variables to three using Kirchoff's point rule.

Q3. (a) Consider the system of non-homogenous linear algebraic equations,

$$\begin{aligned} ax_1 \quad\quad\quad\quad + x_3 &= 161 \\ -x_1 \quad + ax_2 \quad\quad\quad &= 302 \\ x_2 \quad + \ x_3 &= 2016 \end{aligned}$$

If there is no parameters in the solution of the consistent system, what are the values that a can **NOT** have? ***[10 points]***

(b) Consider the matrix

$$B = \begin{pmatrix} 3 & 6 & -1 & -5 & 5 \\ 2 & 4 & -1 & -3 & 2 \\ 3 & 6 & -2 & -4 & 1 \end{pmatrix}$$

If the system $BX = C$ is consistent. How many parameters does the solution have? What is the rank of the augmented matrix $(B|C)$?

[10 points]

Q4. (a) Find the eigenvalues of the matrix,

$$A = \begin{pmatrix} 3 & 0 & 0 \\ -4 & 6 & 2 \\ 16 & -15 & -5 \end{pmatrix}$$

[10 points]

(b) Find a vector $\boldsymbol{v}$ such that $A^{302}\boldsymbol{v} = \boldsymbol{v}$. ***[10 points]***

Q5. (a) Consider

$$A = \begin{pmatrix} 0 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}.$$

Find matrices P and D, with P orthogonal and D diagonal, such that $A = PDP^{-1}$. ***[10 points]***

(b) Find the eigenvalues and eigenvectors of A^{-1}. ***[5 points]***

Q6. Find the inverse of the matrix A, if it exists, using the augmented matrix method where

$$A = \begin{pmatrix} 1 & -2 & 2 \\ 3 & 0 & 1 \\ 1 & -1 & 2 \end{pmatrix}$$

Show the step by step calculations for full marks. ***[10 pts]***