King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics

MATH 302, Semester 161 (2016-2017)

EXAM II

December 7, 2016

Allowed Time: 150 mins

Student Name:

Student ID Number:

Section Number:

Instructor's Name:

Instructions:

1. Write neatly and legibly -- *you may lose points for messy work*.
2. Show all your work -- *no points for answers without justification*.
3. Programmable calculators and Mobiles are not allowed.
4. Make sure that you have 4 questions (5 pages + cover page + formula sheet).

Problem No.	Points	Maximum Points
1		25
2		25
3		30
4		20
Total:		**100**

Coordinator: Dr A. N. Duman

Q1. Let $\boldsymbol{A} = 3z^2 \sin\varphi\, \widehat{\boldsymbol{a}}_{\boldsymbol{\rho}} + \rho\cos 2\varphi\, \widehat{\boldsymbol{a}}_{\boldsymbol{\varphi}} - \rho z \widehat{\boldsymbol{a}}_{\boldsymbol{z}}$ at $P(2\sqrt{3}, \frac{\pi}{6}, 2)$.

(a) Determine the vector component of $\boldsymbol{A}$ that is tangential to the surface $\theta = \pi/3$. Give your answer in cylindrical coordinates.

[15 pts]

(b) Determine the angle that $\boldsymbol{A}$ makes the tangent plane of the surface $r = 4$. ***[10 pts]***

Q2.

(a) Find the directional derivative $T = r^2 \sin\theta \cos\varphi$ in the direction $3\hat{\boldsymbol{a}}_x - 4\hat{\boldsymbol{a}}_z$ at the point $P(1, \frac{\pi}{6}, \frac{\pi}{2})$. **[10 pts]**

(b) Find $\boldsymbol{\nabla}^2$V in cylindrical coordinates, where $V = \rho z \cos 2\varphi$, $\rho \neq 0$. **[5 pts]**

(c) Express $\boldsymbol{\nabla}$V in spherical coordinates. **[10 pts]**

Q3. Verify the divergence theorem for the function $\boldsymbol{E} = 2\rho z^2 \hat{\boldsymbol{a}}_{\boldsymbol{\rho}} + \rho \cos^2\varphi\ \hat{\boldsymbol{a}}_{\boldsymbol{z}}$, over region defined by $2 < \rho < 5, -1 < z < 1, 0 < \varphi < 2\pi$. ***[30 points]***

Q4. Let $\boldsymbol{E} = (20\rho\sin\varphi + 6z)\widehat{\boldsymbol{a}}_{\boldsymbol{\rho}} + 10\,\rho\cos\phi\,\widehat{\boldsymbol{a}}_{\varphi} + 6\rho\,\widehat{\boldsymbol{a}}_{\boldsymbol{z}}$ be the electric field on a certain region of space.

(a) Verify that $\boldsymbol{E}$ is a conservative field. ***[5 points]***

(b) Find the electric potential function V. ***[10 points]***

(c) Given two points $A(1,0,1)$ and $B(4,\pi/6,0)$ inside this region, find the electric potential at A, i.e. $V(A)$, given that $V(B) = 5$. ***[5 points]***

Formulae in cylindrical and spherical coordinate systems

Differential of displacement

Cylindrical: $d\boldsymbol{l} = d\rho\, \widehat{\boldsymbol{a}_{\boldsymbol{\rho}}} + \rho d\phi\, \widehat{\boldsymbol{a}_{\boldsymbol{\phi}}} + dz\, \widehat{\boldsymbol{a}_{\boldsymbol{z}}}$

Spherical: $d\boldsymbol{l} = dr\, \widehat{\boldsymbol{a}_{\boldsymbol{r}}} + r d\theta\, \widehat{\boldsymbol{a}_{\boldsymbol{\theta}}} + r \sin\theta\, d\phi\, \widehat{\boldsymbol{a}_{\boldsymbol{\phi}}}$

Gradient of a scalar field, ∇ V

Cylindrical: $\boldsymbol{\nabla}\, \mathrm{V} = \frac{\partial V}{\partial \boldsymbol{\rho}}\, \widehat{\boldsymbol{a}_{\boldsymbol{\rho}}} + \frac{1}{\boldsymbol{\rho}} \frac{\partial V}{\partial \boldsymbol{\phi}}\, \widehat{\boldsymbol{a}_{\boldsymbol{\phi}}} + \frac{\partial V}{\partial \boldsymbol{z}}\, \widehat{\boldsymbol{a}_{\boldsymbol{z}}}$

Spherical: $\boldsymbol{\nabla}\, \mathrm{V} = \frac{\partial V}{\partial \boldsymbol{r}}\, \widehat{\boldsymbol{a}_{\boldsymbol{r}}} + \frac{1}{\boldsymbol{r}} \frac{\partial V}{\partial \boldsymbol{\theta}}\, \widehat{\boldsymbol{a}_{\boldsymbol{\theta}}} + \frac{1}{\boldsymbol{r} \sin\theta} \frac{\partial V}{\partial \boldsymbol{\phi}}\, \widehat{\boldsymbol{a}_{\boldsymbol{\phi}}}$

Divergence of a vector field, ∇ · G

Cylindrical: $\boldsymbol{\nabla} \cdot \mathbf{G} = \frac{1}{\boldsymbol{\rho}} \frac{\partial(\rho G_{\rho})}{\partial \boldsymbol{\rho}} + \frac{1}{\boldsymbol{\rho}} \frac{\partial(G_{\boldsymbol{\phi}})}{\partial \boldsymbol{\phi}} + \frac{\partial(G_z)}{\partial \boldsymbol{z}}$

Spherical: $\boldsymbol{\nabla} \cdot \mathbf{G} = \frac{1}{r^2} \frac{\partial(r^2 G_r)}{\partial \boldsymbol{r}} + \frac{1}{\boldsymbol{r} \sin\theta} \frac{\partial(\sin\theta\, G_\theta)}{\partial\theta} + \frac{1}{\boldsymbol{r} \sin\theta} \frac{\partial(G_{\boldsymbol{\phi}})}{\partial \boldsymbol{\phi}}$

Relationship between Cartesian, Cylindrical and Spherical Coordinates

$$\begin{pmatrix} A_\rho \\ A_\varphi \\ A_z \end{pmatrix} = \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$

$$\begin{pmatrix} A_r \\ A_\theta \\ A_\varphi \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\varphi & \sin\theta\sin\varphi & \cos\theta \\ \cos\theta\cos\varphi & \cos\theta\sin\varphi & -\sin\theta \\ -\sin\varphi & \cos\varphi & 0 \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$