

## Math 440 - 161 Major Exam 1

Q1 a) Find the tangent space to the cone with equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0 \text{ at the origin and at any other point.}$$

b) Give a parametrization of this cone and write down the first fundamental form in these coordinates.

Q2 a) Give a parametrization of a surface of revolution and write down the geodesic equation in these coordinates.

b) Prove that all great circles on a sphere are geodesics.

You may use the existence and uniqueness theorem for a system of ODEs with prescribed initial conditions.

c) Find the area of a lune – the region on a sphere bounded by two great half circles from the north pole to the south pole .

Use this to find the area of a lune given by two great half circles passing through a point and its antipodal point.

Q3 a) Using the Serret-Frenet formulae, show that a space curve is uniquely determined - up to rigid motions - by its torsion and curvature.

b) Show - using the definition of length of curves - that the line segment from a point  $P$  to a point  $Q$  in  $\mathbb{R}^3$  is the curve of smallest length joining  $P$  and  $Q$ .