Math 440-161 Final Exam

Name:____________________ id#:_________________

Write answers so that they are readable. No credit for answers that I cannot read.

Q1: a)Give a precise definition of the following concepts:

(i) A smooth 2 dimensional manifold
(ii) The differential of a differentiable function $f: S_1 \to S_2$, where S_1 and S_2 are surfaces in $\mathbb{R}^3$
b) Let S be the unit sphere of radius 1, centered at the origin. Set up a 1:1 correspondence f between points on the tangent plane at the south pole and the sphere minus its north pole.
c) Compute the differential of f at any point $(a, b, -1)$ of the tangent plane to the sphere at the south pole.

Q2: Realize the hyperboloid of 1 sheet as a surface of revolution . Show that through every point of such a hyperboloid, there pass two straight lines which lie of the hyperboloid.

Q3: Show that $sl(2, \mathbb{R})$ - the set of all 2×2 matrices of determinant 1- is a three dimensional manifold.

This means that you have to cover $sl(2, \mathbb{R})$ by open sets , where each open set is in 1:1 correspondence with an open set in $\mathbb{R}^3$, and these mappings are differentiable with a differentiable inverse.

Q4) (a) Let $F(x, y, z)$ and $G(x, y, z)$ be a differentiable functions. Prove that if a local maximum or minimum of F on a level set of G is achieved at a point p where gradient of G is non-zero, then necessarily $grad(F)(p) = \lambda grad(G)(p)$ for some constant λ.

(b) Use (a) to write down the equation whose roots give the maximum and minimum values of normal curvatures.

Q5) (a) Given a 1 parameter subgroup of transformations $g(t)$, it defines a vector field by assigning to every point p the tangent vector at p to the curve $c(t) = g(t)p$.

Write down explicitly the vector fields given by rotations about the coordinate axes.

If these vector fields are , say X_i , $i = 1,2,3$ then each of these fields defines an operator

on differentiable functions , namely $D_{X_i}(f) = grad(f).X_i$
Compute the brackets $[D_{X_1}, D_{X_2}]$, $[D_{X_2}, D_{X_3}]$, $[D_{X_3}, D_{X_1}]$.