

Math 440 Differential Geometry Syllabus (Semester 161)

Differential Geometry is the study of geometric questions by means of Calculus. This course introduces the study of curves and surfaces in Euclidean spaces.

- The geometry of curves involves the concept of torsion (the twisting out of a plane) and curvature (the bending away from a line).
- The geometry of surfaces involves the concepts of mean and Gaussian curvature (the bending away from a plane).

The main concepts we will study are

- Parameterization of curves and surfaces
- Frenet frames
- Curvature and torsion of space curves
- Geometry of surfaces
- Different types of curvatures of surfaces
- First and second fundamental forms
- Fundamental equations of surfaces, Fundamental theorem of surfaces
- Geodesics and some of their properties
- Isometries

These will also lay foundations to introduce the notion of abstract surfaces (2-dimensional manifolds) and manifolds at the end of the course.

2. TEXTBOOK

Differential Geometry and its Applications by John Oprea, The Mathematical Association of America, 2nd edition (2007).

3. TOPIC-WISE BREAK-UP

2/2

| Week | Sec. | Topics |
|------|----------------|---|
| 1 | 1.1 | Introduction (The Geometry of Curves) |
| 2&3 | 1.2 1.3 1.4 | Arclength Parameterization Frenet Formulas Non-Unit Speed Curves |
| 4 | 1.5 1.7 | Some Implications of Curvature and Torsion The Geometry of Curves and Maple |
| 5&6 | 2.1 2.2 | Introduction (Surfaces) The Geometry of Surfaces |
| 7 | 2.3 2.4 | The Linear Algebra of Surfaces Normal Curvature |
| 8 | 2.5 | Surfaces and Maple |
| 9 | 3.1 3.2 | Introduction (Curvatures) Calculating Curvature |

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| 10 | 3.3 3.4 | Surfaces of Revolution A Formula for Gauss Curvature |
| 11 | | First and second fundamental forms Fundamental equations of surfaces Fundamental theorem of surfaces |
| 12 & 13 | 5.1 5.2 | Introduction (Geodesics, Metrics and Isometries) The Geodesic Equation and the Clairaut Relation |
| 14 & 15 | | Isometries Introduction to Manifolds and Riemannian Geometry |

4. SOFTWARE

The textbook contains sections where the notions of geometry of curves and surfaces are demonstrated (graphically & computationally) using the software Maple. We will use Maple mostly as a visual tool to understand the concepts intuitively. No prior knowledge of Maple is required.

5. GRADING

HW: 15%

Project 5%

Major Exam: 30%

Final Exam: 50%

6. REFERENCE BOOKS

- Elementary Differential Geometry, by Andrew Pressley. Springer (2002).
- Diff. Geometry of Curves & Surfaces, by Manfredo Do Carmo.
- Modern Differential Geometry of Curves and Surfaces with Mathematica by A.

Gray.