Use Laplace transform to solve
\[
\frac{1}{\gamma} \frac{\partial}{\partial \gamma} \left( \gamma \frac{\partial u}{\partial \gamma} \right) + \frac{\partial^2 u}{\partial z^2} = 0, \quad 0 < z < a, \quad 0 < \gamma < \infty.
\]

with the following conditions

\[
u(\gamma, 0) = 1, \quad u(a, z) = 0,
\]

\[
\lim_{{z \to 0}} |u(\gamma, z)| < \infty,
\]

\[
\lim_{{\gamma \to 0}} |u(\gamma, z)| < \infty.
\]