Exercise 1
Find all the singularities of the following functions, and describe their nature

(a) \( \frac{1}{z^2(e^z - 1)} \)

(b) \( \frac{1}{\cos(1/z)} \)
Exercise 2

Assume that $f$ has an isolated singularity at 0 and $\lim_{z \to 0} |z|^{2/3} |f(z)| = 0$. Show that 0 is a removable singularity of $f$. 
Exercise 3
Find all analytic functions on $\hat{\mathbb{C}}$ with a pole of order 2 at 0.
Exercise 4

Find the Laurent expansion of

\[ f(z) = \frac{1}{z(z - 1)(z - 2)} \]

(in powers of \( z \)) for

(a) \( 0 < |z| < 1 \)
(b) \( 1 < |z| < 2 \)
Exercise 5

Let \( \{n_1, n_2, \ldots, n_k\} \) be a set of positive integers and

\[
R(z) = \frac{1}{(z^{n_1} - 1)(z^{n_2} - 1) \ldots (z^{n_k} - 1)}
\]

1. Find \( a_{-k} \) the coefficient in the Laurent expansion for \( R \) about \( z = 1 \).
2. Find \( a_{-n} \) for every \( n > k \).
Exercise 6
Suppose that $f$ is entire and $|f(z)| = 1$ on $|z| = 1$. Prove that

$$f(z) = Cz^n$$

(Hint: first use the maximum and the minimum modulus theorem to show that

$$f(z) = C \prod_{k=1}^{n} \left( \frac{z - a_k}{1 - \overline{a_k}z} \right)$$

)
Exercise 7

Suppose that $|f(z)| \leq 1$ for $|z| < 1$ and $f$ is analytic. By considering the function $g : \Delta \to \Delta$ defined by $g(z) = \frac{f(z) - a}{1 - \bar{a}f(z)}$ where $a = f(0)$. Prove that

$$\frac{|f(0)| - |z|}{1 - |f(0)||z|} \leq |f(z)| \leq \frac{|f(0)| + |z|}{1 + |f(0)||z|}$$

for $|z| < 1$. 