King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
MATH533 - Complex Variables
Exam II – Semester 161

Exercise 1

Find all the singularities of the following functions, and describe their nature

(a) $\dfrac{1}{z^2(e^z - 1)}$

(b) $\dfrac{1}{\cos(1/z)}$

Exercise 2

Assume that f has an isolated singularity at 0 and $\lim_{z \to 0} |z|^{2/3} |f(z)| = 0$. Show that 0 is a removable singularity of f.

Exercise 3

Find all analytic functions on $\hat{\mathbb{C}}$ with a pole of order 2 at 0.

Exercise 4

Find the Laurent expansion of

$$f(z) = \frac{1}{z(z-1)(z-2)}$$

(in powers of z) for

(a) $0 < |z| < 1$

(b) $1 < |z| < 2$

Exercise 5

Let $\{n_1, n_2, \ldots, n_k\}$ be a set of positive integers and

$$R(z) = \frac{1}{(z^{n_1} - 1)(z^{n_2} - 1) \ldots (z^{n_k} - 1)}$$

1. Find a_{-k} the coefficient in the Laurent expansion for R about $z = 1$.
2. Find a_{-n} for every $n > k$.

Exercise 6

Suppose that f is *entire* and $|f(z)| = 1$ on $|z| = 1$. Prove that

$$f(z) = Cz^n$$

(Hint : first use the maximum and the minimum modulus theorem to show that $f(z) = C\prod_{k=1}^{n}\left(\frac{z-\alpha_k}{1-\overline{\alpha_k}z}\right)$)

Exercise 7

Suppose that $|f(z)| \leq 1$ for $|z| < 1$ and f is analytic. By considering the function $g : \Delta \to \Delta$ defined by $g(z) = \dfrac{f(z) - a}{1 - \overline{a} f(z)}$ where $a = f(0)$. Prove that

$$\frac{|f(0)| - |z|}{1 - |f(0)||z|} \leq |f(z)| \leq \frac{|f(0)| + |z|}{1 + |f(0)||z|}$$

for $|z| < 1$.