Q.No.1: (2+2+3 = 7 points) Suppose that we want to fit the model $y = \beta x + \epsilon$ using weighted least squares. Assume that the observations are uncorrelated but have unequal variance.

(a) Find a general formula for the weighted least-square estimator of $\beta$. Simplify as much as possible.

(b) What is the variance of the weighted least-square estimator derived in part (a)?
Suppose that \( Var(y_i) = cx_i \) i.e. the variance is proportional to the corresponding \( x_i \). Using the result of parts (a) and (b), find the weighted least-square estimator of \( \beta \) and the variance of this estimator. (Hint: The weights \( (w_i) \) should be reciprocal of \( x_i \).)
Q.No.2:- (4 points) Suppose that the studentized PRESS residual $t_i = \frac{e(i)}{\sqrt{\text{Var}(e(i))}}$ where $e(i) = \frac{e_i}{1-h_{ii}}$.

Mathematically show that $t_i = \frac{e_i}{\sqrt{\sigma^2(1-h_{ii})}}$. 
Q.No.3: (4 points) Consider the multiple linear regression model $y = X\beta + \epsilon$. Show that the least-square estimator of $\beta$ (i.e. $\hat{\beta} = (X'X)^{-1}X'y$) can be written as $\hat{\beta} = \beta + X^{-1}H\epsilon$ where $H = X(X'X)^{-1}X'$. 

With the Best Wishes