King Fahd University of Petroleum & Minerals

Second Major Examination

Faculty: Science	**Department: Mathematics**
Semester: 171	**Course Name: Actuarial Risk & Credibility Theory**
Instructor: Abedalhay Elmughrabi	**Course No: AS 483**
Exam Date: April 24th, 2017	**Exam Time: 03:30 PM – 05:30 PM**

Student Name:	**ID No.:**

Question No.	Question Full Marks	Question Obtained Marks
1	10 points	
2	10 points	
3	10 points	
4	10 points	
5	10 points	
6	10 points	
7	10 points	
8	10 points	
9	10 points	
10	10 points	
Total	100	Obtained Total:

Exam Instructions

1. Fill in all information required.
2. The exam is composed of **10** questions.
3. Only the following is allowed to be on your desk: SOA approved calculator, pen/pencil, eraser, and sharpener.
4. Calculators cannot be exchanged during the examination.
5. No use of smart devices with communications capabilities (mini laptops, pens, watches, phones, etc.)
6. Cell phones must be turned off and placed under your bench facedown.
7. No questions are allowed during the exam.
8. All material related to the course should be put away
9. Final correct answers have significant weights
10. Answers without calculations/steps will receive zero marks.
11. Be clean, neat and tidy, else your work may not be marked
12. Students must not communicate with one another in any manner whatsoever during the examination.

GOOD LUCK

Questions 1 (10 Points):

An insurer has excess-of-loss reinsurance on auto insurance. You are given:
(i) Total expected losses in the year 2001 are 10,000,000.
(ii) In the year 2001 individual losses have a Pareto distribution with

$$F(x) = 1 - \left(\frac{2000}{x + 2000}\right)^2, x > 0$$

(iii) Reinsurance will pay the excess of each loss over 3000.
(iv) Each year, the reinsurer is paid a ceded premium, Cyear; equal to 110% of the expected losses covered by the reinsurance.
(v) Individual losses increase 5% each year due to inflation.
(vi) The frequency distribution does not change.
(a) Calculate C_{2001}
(b) Calculate $\frac{C_{2002}}{C_{2001}}$

Questions 2 (10 Points):

For an aggregate loss distribution S:

(i) The number of claims has a negative binomial distribution with $r = 16$ and $\beta = 6$.
(ii) The claim amounts are uniformly distributed on the interval (0,8).
(iii) The number of claims and claim amounts are mutually independent.

Using the normal approximation for aggregate losses, calculate the premium such that the probability that aggregate losses will exceed the premium is 5%.

Questions 3 (10 Points):

You are given:

(i) A sample $x_1, x_2, \dots, x_{10}$ is drawn from a distribution with probability density function:

$$\frac{1}{2}\left[\frac{1}{\theta}\exp\left(-\frac{x}{\theta}\right)+\frac{1}{\sigma}\exp\left(-\frac{x}{\sigma}\right)\right], \qquad 0 < x < \infty$$

(ii) $\theta > \sigma$

(iii) $\sum x_i = 150$ and $\sum x_i^2 = 5000$

Estimate θ by matching the first two sample moments to the corresponding population quantiles.

Questions 4 (10 Points):

A compound Poisson distribution has $\lambda=5$ and claims amount distribution as follows

x	P(x)
100	0.8
500	0.16
1000	0.04

Calculate the probability that aggregate claims will be exactly 600?

Questions 5 (10 Points):

S_1 has a compound Poisson distribution with parameter λ_1=1 and discrete claim amount distribution $P_1(x)$: $P_1(1)$=0.75 and $P_1(5)$=0.25.
S_2 has a compound Poisson distribution with parameter λ_2=1 and discrete claim amount distribution $P_2(x)$: $P_2(1)$=0.5 and $P_2(5)$=0.5.
S_1 and S_2 are independent. Determine $P(S_1+S_2 \leq 3)$

Questions 6 (10 Points):

You are given:

- Losses follow a Burr distribution with $\alpha = 2$. A random sample of 15 losses is:

  195 255 270 280 350 360 365 380 415 450 490 550 575 590 615

- The parameters γ and θ are estimated by percentile matching using the smoothed empirical estimates of the 30th and 65th percentiles.

  Calculate the estimate of γ.

Questions 7 (10 Points):

Aggregate claims S has a compound Poisson aggregate claim distribution with discrete individual claim amount distribution $f_X(1) = \frac{1}{3}$ and $f_X(3) = \frac{2}{3}$.
You are also given $f_S(4) = f_S(3) + 6f_S(1)$. Determine Var (S).

Questions 8 (10 Points):

For an aggregate claims S, you are given:

(i) S can assume values that are multiple of 10.

(ii) $E[(S-10)_+]=0.6$ and $E[(S-20)_+]=0.2$

Find $F_S(10)$

Questions 9 (10 Points):

$$F(x) = \begin{cases} 0, & x < 0 \\ 1 - 0.3e^{-0.00001x}, & x \geq 0 \end{cases}$$

For the distribution above, determine the following:

- The loss elimination ratio with an ordinary deductible of 5,000.
- The effect of inflation at 10% on an ordinary deductible of 5,000 applied to the distribution.

Questions 10 (10 Points):

The number of claims in a period has a geometric distribution with mean 3. The amount of each claim X follows $\Pr(X = x) = 0.25$; $x = 1, 2, 3, 4$: The number of claims and the claim amounts are independent. S is the aggregate claim amount in the period.

Calculate $F_S(3)$: