King Fahd University of Petroleum & Minerals

Final Examination

Faculty: Science	**Department: Mathematics**
Semester: 171	**Course Name: Actuarial Risk & Credibility Theory**
Instructor: Abedalhay Elmughrabi	**Course No: AS 483**
Exam Date: May 22nd, 2017	**Exam Time: 8:00 AM – 11:00 AM**

Student Name:	**ID No.:**

Question No.	Question Full Marks	Question Obtained Marks	Question No.	Question Full Marks	Question Obtained Marks
1	9 points		11	8 points	
2	8 points		12	9 points	
3	9 points		13	7 points	
4	8 points		14	7 points	
5	9 points		15	8 points	
6	9 points		16	7 points	
7	9 points		17	7 points	
8	9 points				
9	8 points				
10	9 points				

Obtained Total:

/ 140

Exam Instructions

1. Fill in all information required.
2. The exam is composed of **17** questions.
3. Only the following is allowed to be on your desk: SOA approved calculator, pen/pencil, eraser, and sharpener.
4. Calculators cannot be exchanged during the examination.
5. No use of smart devices with communications capabilities (mini laptops, pens, watches, phones, etc.)
6. Cell phones must be turned off and placed under your bench facedown.
7. No questions are allowed during the exam.
8. All material related to the course should be put away
9. Final correct answers have significant weights
10. Answers without calculations/steps will receive zero marks.
11. Be clean, neat and tidy, else your work may not be marked
12. Students must not communicate with one another in any manner whatsoever during the examination.

GOOD LUCK

<u>Questions 1 (9 Points):</u> You are simulating the gain/loss from insurance where:

(i) Claim occurrences follow a Poisson process with = $\frac{2}{3}$ per year.
(ii) Times between successive claims follow an exponential distribution with Mean 1.5
(iii) Each claim amount is 1, 2 or 3 with p(1) = 0.25, p(2) = 0.25, and p(3) = 0.50
(iv) Claim occurrences and amounts are independent. Successive time claims are independent.
(v) The annual premium equals expected annual claims plus 1.8 times the standard deviation of annual claims.
(vi) i = 0.

You use 0.25, 0.40, 0.60, and 0.80 from the unit interval and the inversion method to simulate time between claims.

You use 0.30, 0.60, 0.20, and 0.70 from the unit interval and the inversion method to simulate claim size.

Calculate the gain or loss from the insurer's viewpoint during the first 2 years from this simulation.

Questions 2 (8 Points):

Let S be a compound Poisson distribution with parameter λ= 0.04 and individual claim distribution given by

x	$f_X(x)$
1	0.5
2	0.4
3	0.1

Show that:

$$f_S(n) = \frac{1}{n}[0.02 f_S(n-1) + 0.032\, f_S(n-2) + 0.012\, f_S(n-3)]$$

Questions 3 (9 Points):

Personal auto property damage claims in a certain region are known to follow the Weibull distribution:

$$F(x) = 1 - e^{-\left(\frac{x}{\theta}\right)^{0.2}}, \qquad x>0$$

A sample of four claims is:

130 240 300 540

The values of two additional claims are known to exceed 1000.

Determine the maximum likelihood estimate of θ.

Questions 4 (8A Points):

Prior to observing any claims, you believed that claim sizes followed a Pareto distribution with parameters $\theta = 10$ and $\alpha = 1$, 2 or 3, with each value being equally likely.

You then observe one claim of 20 for a randomly selected risk.

Determine the posterior probability that the next claim for this risk will be greater than 30.

Questions 5 (9 Points):

You are given:

(i) The number of claims has probability function:

$$p(x) = \binom{m}{x} q^x (1-q)^{m-x}, \qquad x = 0,1,2,\ldots m$$

(ii) The actual number of claims must be within 1% of the expected number of claims with probability 0.95.

(iii)

(iv) The expected number of claims for full credibility is 34,574.

Determine q.

Questions 6 (9 Points):
For a portfolio of insurance risks, aggregate losses per year per exposure follow a normal distribution with mean θ and standard deviation 1000, with θ varying by class as follows:

Class	θ	Percent of Risks in Class
X	2000	60%
Y	3000	30%
Z	4000	10%

A randomly selected risk has the following experience over three years:

Year	Number of Exposures	Percent of Risks in Class
1	24	24,000
2	30	36,000
3	26	28,000

Calculate the Bühlmann-Straub estimate of the mean aggregate losses per year per exposure in Year 4 for this risk.

Questions 7 (9 Points):

For a portfolio of independent risks, the number of claims for each risk in a year follows a Poisson distribution with means given in the following table:

Class	Mena Number of Claims per Risk	Number of Risks
1	1	900
2	10	90
3	20	10

You observe x claims in Year 1 for a randomly selected risk.

The Bühlmann credibility estimate of the number of claims for the same risk in Year 2 is 11.983.

Determine x.

Questions 8 (9 Points):

The partial credibility approach is applied to a data of 50 claim amounts. It is assumed that the claim amount distribution is uniform on the interval [0,θ]. The full credibility standard is to be within 5% of the expected claim amount 90% of the time. The partial credibility factor Z is found. After 25 additional claim amounts are recorded, the claim amount distribution is revised to be uniform on the interval [0,1.2θ]. The revised partial credibility factor $Z^*$is found. Fin the ratio $\frac{Z}{Z^*}$

Questions 9 (8 Points):

You are given the following probability density function:

$$f(x) = \frac{x^2}{9} 1_{[0 \le x \le 3]}$$

You are to simulate three observations from the distribution using the inversion method and the following three random number from the uniform distribution on (0,1): 0.008, 0.729, 0.125. Using the three simulated observations, estimate the mean of the distribution.

Questions 10 (9 Points):

You are given:

(i) The cumulative distribution for the annual number of losses for a policyholder is:

n	$F_N(n)$
0	0.125
1	0.312
2	0.500
3	0.656
4	0.773
5	0.855
.	
.	
.	

(ii) The loss amounts follow the Weibull distribution with $\theta = 200$ and $\tau = 2$.

(iii) There is a deductible of 150 for each claim subject to an annual maximum out-of pocket of 500 per policy.

The inversion method is used to simulate the number of losses and loss amounts for a policyholder.

(a) For the number of losses use the random number 0.7654.

(b) For loss amounts use the random numbers: 0.2738 0.5152 0.7537 0.6481 0.3153

Use the random numbers in order and only as needed. Based on the simulation, calculate the insurer's aggregate payments for this policyholder.

Questions 11 (8 Points):

You own a fancy light bulb factory. Your workforce is a bit clumsy they keep dropping boxes of light bulbs. The boxes have varying numbers of light bulbs in them, and when dropped, the entire box is destroyed. You are given:

(i) Expected number of boxes dropped per month : 50
(ii) Variance of the number of boxes dropped per month: 100
(iii) Expected value per box: 200
(iv)Variance of the value per box: 400

You pay your employees a bonus if the value of light bulbs destroyed in a month is less than 8000.

Assuming independence and using the normal approximation, calculate the Probability that you will pay your employees a bonus next month.

Questions 12 (9 Points):

For a portfolio of independent risks, you are given:

(i) The risks are divided into two classes, Class A and Class B.
(ii) Equal numbers of risks are in Class A and Class B.
(iii) For each risk, the probability of having exactly 1 claim during the year is 20% and the probability of having 0 claims is 80%.
(iv) All claims for Class A are of size 2.
(v) All claims for Class B are of size c, an unknown but fixed quantity.

One risk is chosen at random, and the total loss for one year for that risk is observed. You wish to estimate the expected loss for that same risk in the following year.

Determine the limit of the Bühlmann credibility factor as c goes to infinity

Questions 13 (7 Points):

Let $X_1, \ldots, X_n$ be a random sample with a Poisson distribution with mean θ and let θ be Gamma (α,β) with density

$$g(\theta) = \frac{\theta^{\alpha-1} e^{-\beta\theta} \beta^{\alpha}}{(\alpha-1)!} \quad \theta > 0$$

Find the Bayes estimator of θ under the square error loss?

Questions 14 (7 Points):

Let X_1 be a Pareto random variable with parameters $\alpha = 2$ and $\theta = 100$.
Let X_2 be a random variable with Uniform distribution on (0, 864): Find p such that

$$\frac{TVaR_{0.99}(X_1)}{VaR_{0.99}(X_1)} = \frac{TVaR_p(X_2)}{VaR_p(X_2)}$$

Questions 15 (8 Points):

A group dental policy has a negative binomial claim count distribution with mean 300 and variance 800.

Ground-up severity is given by the following table:

Severity	Probability
40	0.25
80	0.25
120	0.25
200	0.25

You expect severity to increase 50% with no change in frequency. You decide to impose a per claim deductible of 100.

Calculate the expected total claim payment after these changes.

Questions 16 (7 Points):

Let N be a counting distribution in C(a,b,0) satisfying:

- $P_0 = \frac{1}{4^5}$
- $\frac{p_k}{p_{k-1}} = c\left(0.25 + \frac{1}{k}\right), \ \ k = 1,2,3, \ldots$

Determine the value of c?

Questions 17 (7 Points):

Let X have a Burr distribution with parameters α=1, γ=2, and $\theta=\sqrt{1{,}000}$ and let Y have a Pareto distribution with parameters α=1 and θ=1,000. Let Z be a mixture of X and Y with equal weight on each component. Determine the median of Z.