1. Let \( h(x) = \frac{g(x)}{f(x) + g(x)} \). If \( f(4) = 1 \), \( g(4) = 2 \), \( f'(4) = 3 \), and \( g'(4) = -3 \), then \( h'(4) = \)

(a) -1
(b) 0
(c) -3
(d) 3
(e) -2

2. A particle moves in a straight line and has acceleration given by \( a(t) = 4t + 10 \). Its initial velocity is \( v(0) = -3 \) feet/sec and its initial displacement \( s(0) = 5 \) feet, its position is given by

(a) \( s(t) = \frac{2}{3} t^3 + 5t^2 - 3t + 5 \)
(b) \( s(t) = \frac{2}{3} t^5 + 5t^2 + 3t - 5 \)
(c) \( s(t) = 3t^3 + 10t^2 - 3t + 5 \)
(d) \( s(t) = 3t^3 + 10t^2 + 3t - 5 \)
(e) \( s(t) = \frac{2}{3} t^3 + 5t^2 - 3t \)
3. Let \( f(x) = cx + \ln(\cos x) \) where \( c \) is a constant. The value of \( c \) such that \( f'(\frac{\pi}{4}) = 6 \) equals to

(a) 7
(b) 6
(c) −2
(d) 1
(e) 0

4. \( \tanh(\ln x) = \)

(a) \( \frac{x^2 - 1}{x^2 + 1} \)
(b) \( \frac{x^2 + 1}{x^2 - 1} \)
(c) \( \frac{\ln x^2 - 1}{\ln x^2 + 1} \)
(d) \( \frac{x - 1}{x + 1} \)
(e) 0
5. Suppose $f''$ is continuous on $(-\infty, \infty)$. If $f'(2) = 0$ and $f''(2) = -5$ then

(a) $f$ has a local maximum at $x = 2$

(b) $f$ has a local minimum at $x = 2$

(c) $f$ has a point of inflection at $x = 2$

(d) $f$ is increasing at $x = 2$

(e) $f$ is concave upward at $x = 2$

6. The sum of all positive real numbers $a$, that makes the function

$$f(x) = \begin{cases} 
ax + 3 & \text{if } x > a \\
\frac{x^2 - x + 2a^2}{a} & \text{if } x \leq a
\end{cases}$$

continuous everywhere, is

(a) $\frac{3}{2}$

(b) 1

(c) $\frac{5}{2}$

(d) 2

(e) 3
7. If \( \cosh x = \frac{5}{3} \) and \( x > 0 \), then

(a) \( \tanh x = \frac{4}{5} \)

(b) \( \coth x = \frac{3}{5} \)

(c) \( \sinh x = \frac{3}{4} \)

(d) \( \text{csch} \, x = \frac{4}{3} \)

(e) \( \text{sech} \, x = 1 \)

8. Let \( f(x) = x\sqrt{9 - x^2} \) on the interval \([-3, 3]\). The function \( f \) attains its absolute maximum value at

(a) \( x = \frac{3}{\sqrt{2}} \)

(b) \( x = 3 \)

(c) \( x = -\frac{3}{\sqrt{2}} \)

(d) \( x = -3 \)

(e) \( x = 0 \)
9. The critical number(s) for the function \( f(x) = x^2 \ln x \)
is(are)

(a) \( x = \frac{1}{\sqrt{e}} \)

(b) \( x = 1 \)

(c) \( x = \sqrt{e} \)

(d) \( x = \frac{1}{e} \)

(e) \( x = 1 \) and \( \sqrt{e} \)

10. \( \lim_{x \to 0} \left[ \frac{\sqrt{1 + 2x} - \sqrt{1 - 4x}}{x} \right] = \)

(a) 3

(b) 6

(c) 2

(d) 1

(e) 0
11. \( \lim_{x \to 1^+} \left[ \ln(x^8 - 1) - \ln(x^4 - 1) \right] = \)

(a) \( \ln 2 \)
(b) \( \ln \left( \frac{1}{2} \right) \)
(c) \( \ln 32 \)
(d) \( \ln \left( \frac{1}{32} \right) \)
(e) 0

12. If \( y = \ln (1 + \ln x) \) and \( x > e \), then \( y'' = \)

(a) \( \frac{-2 - \ln x}{x^2(1 + \ln x)^2} \)
(b) \( \frac{-1}{x(1 + \ln x)} \)
(c) \( \frac{1}{x(1 + \ln x)} \)
(d) \( \frac{2 + \ln x}{x^2(1 + \ln x)^2} \)
(e) \( \frac{1}{1 + \ln x} \)
13. The area of the largest rectangle that can be inscribed in a circle of radius 1 is

(a) 2
(b) \( \frac{2}{\sqrt{2}} \)
(c) 4
(d) \( 2 \sqrt{2} \)
(e) \( 2 \pi \)

14. If \( y \sec x = x \tan y + 1 \), then \( \frac{dy}{dx} \) when \( x = 0 \) equals

(a) \( \tan 1 \)
(b) \( \frac{\sec 1}{\tan 1} \)
(c) \( \sec 1 \)
(d) \( \frac{1 + \tan 1}{\sec 1} \)
(e) 1
15. The linearization $L(x)$ of the function $f(x) = \sin x$ at $a = \pi/6$ is

(a) $L(x) = \frac{1}{2} + \frac{\sqrt{3}}{2} \left( x - \frac{\pi}{6} \right)$

(b) $L(x) = \frac{\sqrt{3}}{2} + \frac{1}{2} \left( x - \frac{\pi}{6} \right)$

(c) $L(x) = 1 + \left( x - \frac{\pi}{6} \right)$

(d) $L(x) = 1 + x$

(e) $L(x) = x - \frac{\pi}{6}$

16. If $y = \sqrt{\sin x + y}$, then $\frac{dy}{dx} =$

(a) $\frac{\cos x}{2y - 1}$

(b) $\frac{\cos x}{2y}$

(c) $\frac{\sin x}{2y - 1}$

(d) $\frac{\cos x}{1 - y}$

(e) $\frac{\cos x}{1 - 2y}$
17. Which one of the following statements is **TRUE**?

(a) The derivative of a rational function is a rational function

(b) If \( f \) and \( g \) are differentiable, then \( \frac{d}{dx}[f(x)g(x)] = f'(x)g'(x) \)

(c) If \( y = e^2 \), then \( y' = 2e \)

(d) \( \frac{d}{dx} (10^x) = x \cdot 10^{x-1} \)

(e) \( \frac{d}{dx} (\tan^{-1}(2x)) = \frac{2}{1-2x} \)

18. The slope of the tangent line to the graph of \( y = (x^2+1)^3 \cdot e^{x^2} \) at \( x = 1 \) equals

(a) \( 40e \)

(b) \( 5 \)

(c) \( 20e \)

(d) \( 8e \)

(e) \( 5e \)
19. If \( f(4) = -1 \) and \( f'(x) \geq 5 \) for \( 2 \leq x \leq 4 \), then the largest possible value of \( f(2) \) is

(a) \(-11\)
(b) \(-1\)
(c) \(5\)
(d) \(-\frac{1}{5}\)
(e) \(-5\)

20. The graph of the derivative \( f' \) of a continuous function \( f \) is shown below, then which one of the following statements is TRUE?

![Graph of f']

(a) \( f \) has a local maximum at \( x = 2 \) and \( x = 6 \)
(b) \( f \) is increasing on the intervals \((0, 2), (4, 6), \) and \((6, 8)\)
(c) \( f \) has a local minimum at \( x = 4 \) and \( x = 6 \)
(d) \( f \) is decreasing on the intervals \((0, 2), (4, 6), \) and \((6, 8)\)
(e) \( f \) has an inflection point at \( x = 6 \)
21. Using a linear approximation (or differentials), the best estimation to $\sqrt[3]{1001}$ is $10 + B$. Then $B =$

(a) $\frac{1}{300}$

(b) $\frac{3}{10}$

(c) $\frac{1}{3}$

(d) $\frac{5}{300}$

(e) $\frac{1}{100}$

22. Let $A$ and $B$ be constants such that $y = A \sin x + B \cos x$ satisfies the equation $y'' + y' - 2y = \sin x$. Then

(a) $A = \frac{-3}{10}$ and $B = \frac{-1}{10}$

(b) $A = \frac{2}{5}$ and $B = \frac{3}{5}$

(c) $A = \frac{2}{3}$ and $B = \frac{5}{2}$

(d) $A = \frac{-1}{3}$ and $B = -1$

(e) $A = 1$ and $B = -1$
23. Which one of the following statements is **FALSE**?

(a) If \( f'(c) = 0 \), then \( f \) has a local maximum or minimum at \( c \)

(b) If \( f \) has a local minimum at \( c \) and \( f''(c) \) exits, then \( f'(c) = 0 \)

(c) If \( f \) is differentiable and \( f(-1) = f(1) \), then there is a number \( c \) such that \( |c| < 1 \) and \( f'(c) = 0 \)

(d) There exists a function \( f \) such that \( f(x) > 0 \), \( f'(x) < 0 \), and \( f''(x) > 0 \) for all \( x \)

(e) If \( f \) and \( g \) are positive increasing functions on an interval \( I \), then \( fg \) is increasing on \( I \)

24. Which one of the following graphs represents the function \( f(x) = \frac{x^2 + 1}{x} \)?
25. Let \( f(x) = x^3 - 6x^2 + 9x + 1 \) and \( x \in [2, 4] \). If \( M \) is the absolute maximum value and \( m \) is the absolute minimum value, then \( M + m = \)

(a) 6  
(b) 8  
(c) 4  
(d) 2  
(e) 1

26. One inflection point of the graph of \( y = e^x \sin x \) on \([-\pi, \pi]\) is

(a) \( \left( \frac{\pi}{2}, e^{\frac{\pi}{2}} \right) \)
(b) \( \left( \frac{\pi}{6}, \frac{e^{\frac{\pi}{6}}}{2} \right) \)
(c) \( \left( \frac{\pi}{3}, \frac{\sqrt{3} e^{\frac{\pi}{2}}}{2} \right) \)
(d) \( (\pi, 0) \)
(e) \( (0, 0) \)
27. The graph of \( y = (1 - x) e^x \) is concave downward on

(a) \((-1, \infty)\)

(b) \((-1, 0)\)

(c) \((-\infty, 1)\)

(d) \((0, 1)\)

(e) \((-e, -1)\)

28. Suppose \( f(x) = \begin{cases} 
2x & \text{for } x \leq 0 \\
2x - 2x^2 & \text{for } 0 < x < 2 \\
2 - 6x & \text{for } x \geq 2
\end{cases} \).

The function \( f \) is not differentiable when

(a) \( x = 2 \)

(b) \( x = 0 \)

(c) \( x \in (0, 2) \)

(d) \( x \in (-\infty, 0] \)

(e) \( x = 0 \) and \( x = 2 \)