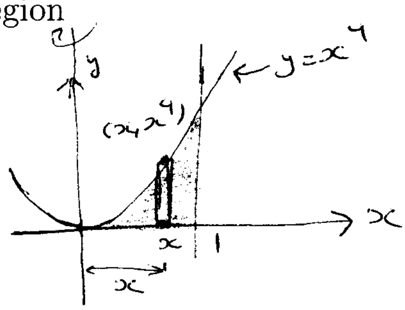
Q	MM	V1	V2	V3	V4
1	a	b	b	b	e
2	a	d	d	c	e
3	a	d	d	e	c
4	a	b	b	a	b
5	a	b	b	c	c
6	a	c	b	e	a
7	a	e	e	a	d
8	a	b	c	b	d
9	a	c	c	d	b
10	a	c	a	c	e
11	a	e	e	d	c
12	a	b	a	c	d
13	a	b	d	a	c
14	a	e	c	d	b
15	a	e	a	d	a
16	a	e	c	a	e
17	a	e	e	b	e
18	a	a	c	e	a
19	a	a	b	e	d
20	a	e	d	a	b

Detailed Solutions →

1. The **volume** of the solid generated by rotating the region bounded by the curves

$$y = x^4,\ y = 0,\ x = 1$$

about the y-axis is



(a) $\frac{\pi}{3}$

(b) $\frac{2\pi}{3}$

(c) π

(d) $\frac{\pi}{2}$

(e) $\frac{2\pi}{5}$

By the Shell method:

$$V = 2\pi \int_0^1 x \cdot x^4 \, dx$$

$$= 2\pi \int_0^1 x^5 \, dx$$

$$= 2\pi \cdot \frac{1}{6} x^6 \Big]_0^1$$

$$= \frac{2\pi}{6} (1-0) = \frac{\pi}{3}$$

2. $\int \sin(3x) \sin(4x) \, dx = \int \frac{1}{2} [\cos(3x-4x) - \cos(3x+4x)] \, dx$

$$= \frac{1}{2} \int \cos(-x) - \cos(7x) \, dx$$

$$= \frac{1}{2} \int \cos x - \cos(7x) \, dx$$

$$= \frac{1}{2} \left[\sin x - \frac{1}{7} \sin(7x) \right] + C$$

$$= \frac{1}{2} \sin x - \frac{1}{14} \sin(7x) + C$$

(a) $\frac{1}{2} \sin x - \frac{1}{14} \sin(7x) + C$

(b) $\frac{1}{2} \cos x - \frac{1}{14} \cos(7x) + C$

(c) $\frac{1}{2} \cos x - \frac{1}{14} \sin(7x) + C$

(d) $\frac{1}{2} \sin x - \frac{1}{14} \cos(7x) + C$

(e) $\frac{1}{2} \sin(7x) - \frac{1}{14} \cos x + C$

3. $\int x\,2^{-x}\,dx =$

(a) $-\dfrac{x\,2^{-x}}{\ln 2} - \dfrac{2^{-x}}{(\ln 2)^2} + C$

(b) $-\dfrac{x\,2^{-x}}{\ln 2} + \dfrac{2^{-x}}{(\ln 2)^2} + C$

(c) $\dfrac{x}{2^{-x}} + \dfrac{\ln 2}{2^{-x}} + C$

(d) $x \ln 2 - \dfrac{2^{-x}}{\ln 2} + C$

(e) $\dfrac{x^2}{\ln 2} - \dfrac{2^{-x}}{\ln 2} + C$

Integration by Parts:

$u = x \qquad dv = 2^{-x}\,dx$

$du = dx \qquad v = -\dfrac{1}{\ln 2}\,2^{-x}$

$$\int \underbrace{x}_{u}\,\underbrace{2^{-x}\,dx}_{dv} = uv - \int v\,du$$

$$= x \cdot \frac{-2^{-x}}{\ln 2} - \int -\frac{2^{-x}}{\ln 2}\,dx$$

$$= -\frac{x\,2^{-x}}{\ln 2} + \frac{1}{\ln 2}\int 2^{-x}\,dx$$

$$= -\frac{x\,2^{-x}}{\ln 2} - \frac{2^{-x}}{(\ln 2)^2} + C$$

$$\begin{array}{r|l} & w - 3 \\ \hline w+3 & w^2 + 3 \\ & w^2 + 3w \\ \hline & -3w + 3 \\ & -3w - 9 \\ \hline & 12 \end{array}$$

4. $\int \dfrac{w^2+3}{w+3}\,dw = \int w - 3 + \dfrac{12}{w+3}\,dw$

$$= \frac{1}{2}w^2 - 3w + 12\ln|w+3| + C$$

(a) $\dfrac{1}{2}\,w^2 - 3\,w + 12\,\ln|w+3| + C$

(b) $2w^2 + 3w + 6\,\ln\,|w+3| + C$

(c) $w^2 - 3w - 6\,\ln|w+3| + C$

(d) $\dfrac{1}{2}\,w^2 + 2\,w - 6\,\ln|w+3| + C$

(e) $2w^2 - 3w - 6\,\ln\,|w+3| + C$

5. The improper integral $\int_8^{\infty} e^{-x/4}\, dx$

$$= \lim_{t\to\infty} \int_8^t e^{-x/4}\, dx$$

$$= \lim_{t\to\infty} -4 e^{-x/4} \Big]_8^t$$

$$= \lim_{t\to\infty} -4 \left(e^{-t/4} - e^{-2}\right)$$

$$= -4\left(0 - e^{-2}\right)$$

$$= 4e^{-2}$$

(a) converges to $4\,e^{-2}$

(b) converges to e^{-2}

(c) converges to 0

(d) converges to $8\,e^{-1}$

(e) diverges

6. The fraction which **does not belong** to the partial fractions decomposition of the rational function

$$\frac{4-x}{(x^4-1)^2} = \frac{4-x}{(x-1)^2(x+1)^2(x^2+1)^2}$$

has the form

$$= \frac{C}{x-1} + \frac{D}{(x-1)^2} + \frac{E}{x+1} + \frac{F}{(x+1)^2} + \frac{Gx+H}{x^2+1} + \frac{Kx+L}{(x^2+1)^2}$$

(a) $\dfrac{Ax+B}{x^2-1}$

(b) $\dfrac{Ax+B}{x^2+1}$

(c) $\dfrac{Ax+B}{(x^2+1)^2}$

(d) $\dfrac{A}{(x-1)^2}$

(e) $\dfrac{A}{(x+1)^2}$

7. $\displaystyle\int \frac{\sqrt{x^2-1}}{x}\,dx =$

$x = \sec\theta, \quad 0 \le \theta < \frac{\pi}{2}$, or $\pi \le \theta < \frac{3\pi}{2}$

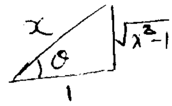
$\displaystyle\int \frac{\sqrt{\sec^2\theta - 1}}{\sec\theta} \cdot \sec\theta \tan\theta \; d\theta$

$\displaystyle\int \frac{\tan\theta}{\sec\theta} \cdot \sec\theta \tan\theta \, d\theta = \int \tan^2\theta \; d\theta$

$\displaystyle = \int \sec^2\theta - 1 \quad d\theta$

$= \tan\theta - \theta + C$

$= \sqrt{x^2-1} - \sec^{-1} x + C$



(a) $\sqrt{x^2-1} - \sec^{-1} x + C$

(b) $\sqrt{x^2-1} - \tan^{-1} x + C$

(c) $\tan x - x + C$

(d) $x\sqrt{x^2-1} - \sec x + C$

(e) $x\sqrt{x^2-1} + \sec^{-1} x + C$

8. The **average value** of $f(y) = \dfrac{\ln y}{\sqrt{y}}$ on the interval $[1, e^2]$ is

$\displaystyle f_{ave} = \frac{1}{e^2-1} \int_1^{e^2} \frac{\ln y}{\sqrt{y}}\, dy = \frac{1}{e^2-1} \int_1^{e^2} y^{-1/2} \ln y \; dy$

$u = \ln y \qquad dv = y^{-1/2}\, dy$

$du = \frac{1}{y}\, dy \quad , \quad v = 2\, y^{1/2}$

$\displaystyle\int y^{-1/2} \ln y \; dy$

$\displaystyle = 2\, y^{1/2} \ln y - 2 \int y^{-1/2}\, dy$

$= 2 y^{1/2} \ln y - 4\, y^{1/2} + C$

(a) $\dfrac{4}{e^2-1}$

(b) $\dfrac{1}{e^2-1}$

(c) $\dfrac{2}{e^2-1}$

(d) $3\,(e^2-1)$

(e) e

$\displaystyle = \frac{1}{e^2-1} \cdot \left[2 y^{1/2} \ln y - 4 y^{1/2} \right]_1^{e^2}$

$\displaystyle = \frac{1}{e^2-1} \left[(2e \cdot 2 - 4e) - (0 - 4) \right]$

$\displaystyle = \frac{1}{e^2-1} \cdot 4 \quad = \frac{4}{e^2-1}$

9. Using the **method of partial fractions**, if

$$\frac{10}{(x-1)(x^2+9)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+9},$$

then $A+B+C=$

$10 = A(x^2+9) + (Bx+C)(x-1)$

$x=1 \Rightarrow 10 = 10A + 0 \Rightarrow \boxed{A=1}$

Coeff of x^2 : $0 = A+B \Rightarrow B = -A \Rightarrow \boxed{B=-1}$

Constant term: $10 = 9A - C \Rightarrow 10 = 9 - C \Rightarrow \boxed{C=-1}$

So $A+B+C = 1-1-1 = -1$

(a) -1

(b) 3

(c) -3

(d) -2

(e) 2

10. Using the substitution $t = \tan(x/2)$, $-\pi < x < \pi$, we obtain

$$\int \frac{1}{2+\sin x - \cos x}\,dx = \int \frac{1}{2 + \frac{2t}{1+t^2} - \frac{1-t^2}{1+t^2}} \cdot \frac{2}{1+t^2}\,dt$$

$$= \int \frac{2}{2(1+t^2)+2t-(1-t^2)}\,dt$$

$$= \int \frac{2}{2+2t^2+2t-1+t^2}\,dt$$

$$= \int \frac{2}{3t^2+2t+1}\,dt$$

(a) $\displaystyle\int \frac{2}{3t^2+2t+1}\,dt$

(b) $\displaystyle\int \frac{2}{(t+1)^2}\,dt$

(c) $\displaystyle\int \frac{2}{3t^2-2t-1}\,dt$

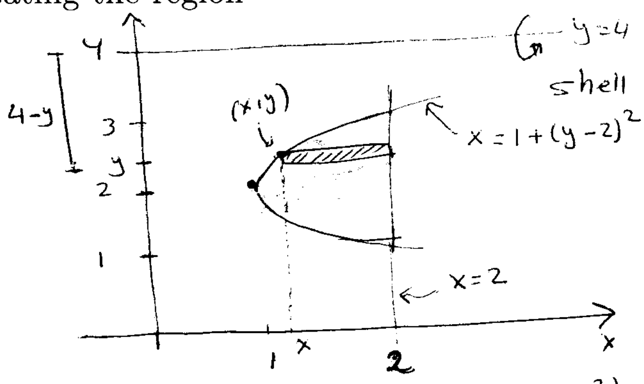
(d) $\displaystyle\int \frac{1}{t^2+2t-1}\,dt$

(e) $\displaystyle\int \frac{1}{2t^2-t+3}\,dt$

11. The **volume** of the solid generated by rotating the region enclosed by the curves

$$x = 1 + (y-2)^2 \text{ and } x = 2$$

about the line $y = 4$ is given by



$2-x = 2-(1+(y-2)^2) = 1-(y-2)^2$

(a) $2\pi \int_1^3 (4-y)\,(1-(y-2)^2)\,dy$

(b) $2\pi \int_1^3 (4-y)\,(1+(y-2)^2)\,dy$

(c) $2\pi \int_1^3 (y-4)\,(2-(y-2)^2)\,dy$

(d) $2\pi \int_1^2 (x-4)\,(2+\sqrt{x-1})\,dx$

(e) $2\pi \int_1^2 (4-x)\,\sqrt{x-1}\,dx$

$V = 2\pi \int_1^3 (4-y)(1-(y-2)^2)\,dy$

$u = \sqrt{x} \Rightarrow du = \frac{1}{2\sqrt{x}}\,dx$

$I = 2\int \cos^5 u\,du = 2\int \cos^4 u \cdot \cos u\,du = 2\int (1-\sin^2 u)^2 \cdot \cos u\,du$

$\boxed{w = \sin u \Rightarrow dw = \cos u\,du}$

$= 2\int (1-w^2)^2\,dw$

$= 2\int 1 - 2w^2 + w^4\,dw$

$= 2\left[w - \frac{2}{3}w^3 + \frac{1}{5}w^5\right] + C$

$= 2\sin u - \frac{4}{3}\sin^3 u + \frac{2}{5}\sin^5 u + C$

$= 2\sin\sqrt{x} - \frac{4}{3}\sin^3\sqrt{x} + \frac{2}{5}\sin^5\sqrt{x} + C$

12. $\int \frac{\cos^5\sqrt{x}}{\sqrt{x}}\,dx =$

(a) $2\sin\sqrt{x} - \frac{4}{3}\sin^3\sqrt{x} + \frac{2}{5}\sin^5\sqrt{x} + C$

(b) $1 - 2\sin^2\sqrt{x} + \sin^4\sqrt{x} + C$

(c) $2\sin\sqrt{x} - 3\sin^2\sqrt{x} + 5\sin^5\sqrt{x} + C$

(d) $2 - \sin^2\sqrt{x} + \frac{1}{5}\sin^5\sqrt{x} + C$

(e) $\sin\sqrt{x} - \frac{1}{3}\sin^3\sqrt{x} + \frac{1}{5}\sin^5\sqrt{x} + C$

13. If the **average value** of $f(x) = 12x^2 + 10x + 5$ on the interval $[a, 0]$ is 4, then the **sum** of all such numbers a is

Clearly, $a<0$.

$$f_{ave} = \frac{1}{0-a}\int_a^0 12x^2+10x+5\, dx$$

$$4 = \frac{1}{-a}\left[4x^3+5x^2+5x\right]_a^0$$

$$-4a = -4a^3 - 5a^2 - 5a$$

$$0 = 4a^3+5a^2+a$$

$$0 = 4a^2+5a+1 \qquad (a \neq 0)$$

$$0 = (4a+1)(a+1)$$

$$\Rightarrow a=-1,\quad a=-\frac{1}{4}$$

$$\text{Sum} = -1-\frac{1}{4} = \frac{-5}{4}$$

(a) $-\dfrac{5}{4}$

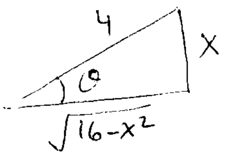
(b) $-\dfrac{3}{2}$

(c) $-\dfrac{3}{4}$

(d) $-\dfrac{5}{8}$

(e) $-\dfrac{3}{8}$

$x = 4\sin\theta,\ -\frac{\pi}{2}<\theta<\frac{\pi}{2}$



$$I = \int \frac{16\sin^2\theta}{\sqrt{16-16\sin^2\theta}}\cdot 4\cos\theta\, d\theta$$

$$= \int \frac{16\sin^2\theta}{4\cos\theta}\cdot 4\cos\theta\, d\theta = 16\int \sin^2\theta\, d\theta$$

$$= 8\int 1-\cos(2\theta)\, d\theta$$

$$= 8\left[\theta - \frac{1}{2}\sin(2\theta)\right] + C$$

$$= 8\theta - 8\sin\theta\cos\theta + C$$

$$= 8\sin^{-1}\left(\frac{x}{4}\right) - 8\cdot\frac{x}{4}\cdot\frac{\sqrt{16-x^2}}{4} + C$$

$$= 8\sin^{-1}\left(\frac{x}{4}\right) - \frac{x}{2}\sqrt{16-x^2} + C$$

14. $\displaystyle\int \frac{x^2}{\sqrt{16-x^2}}\,dx =$

(a) $8\sin^{-1}\left(\dfrac{x}{4}\right) - \dfrac{x}{2}\sqrt{16-x^2} + C$

(b) $4\sin^{-1}\left(\dfrac{x}{4}\right) - x\sqrt{16-x^2} + C$

(c) $8\sin^{-1}(4x) - \dfrac{x}{2}\sqrt{16-x^2} + C$

(d) $16\sin^{-1}\left(\dfrac{x}{4}\right) + \dfrac{x}{2}\sqrt{16-x^2} + C$

(e) $2\sin^{-1}(4x) - 4x\sqrt{16-x^2} + C$

15. The improper integral $\int_{-\infty}^{0} \frac{x^2}{9+x^6}\,dx$ $= \lim_{t\to-\infty} \int_t^0 \frac{x^2}{9+(x^3)^2}\,dx$; $u = x^3$; $du = 3x^2\,dx$

(a) converges to $\frac{\pi}{18}$

(b) converges to $-\frac{\pi}{2}$

(c) converges to $-\frac{\pi}{9}$

(d) converges to $\frac{\pi}{6}$

(e) diverges

$$= \lim_{t\to-\infty} \frac{1}{3}\int_{t^3}^{0} \frac{1}{9+u^2}\,du$$

$$= \lim_{t\to-\infty} \frac{1}{3}\cdot\frac{1}{3}\tan^{-1}\left(\frac{u}{3}\right)\Big]_{t^3}^{0}$$

$$= \lim_{t\to-\infty} -\frac{1}{9}\tan^{-1}\left(\frac{t^3}{3}\right)$$

$$= -\frac{1}{9}\cdot -\frac{\pi}{2} = \frac{\pi}{18}$$

Conv.

16. $\int_{-1/2}^{1} \frac{1}{x^2+x+1}\,dx = \int_{-1/2}^{1} \frac{1}{x^2+x+\frac{1}{4}+\frac{3}{4}}\,dx = \int_{-1/2}^{1} \frac{1}{(x+\frac{1}{2})^2+\frac{3}{4}}\,dx$

(a) $\frac{2\pi}{3\sqrt{3}}$

(b) $\frac{\pi}{\sqrt{3}}$

(c) $\pi\sqrt{3}$

(d) $\frac{\pi}{2\sqrt{3}}$

(e) $\frac{2}{\sqrt{3}}$

$$\boxed{\begin{array}{l} u = x+\frac{1}{2} \Rightarrow du = dx \\ x = -\frac{1}{2} \Rightarrow u = 0 \\ x = 1 \Rightarrow u = \frac{3}{2} \end{array}}$$

$$= \int_0^{3/2} \frac{1}{u^2+3/4}\,du$$

$$= \frac{1}{\sqrt{\frac{3}{4}}}\tan^{-1}\left(\frac{u}{\sqrt{\frac{3}{4}}}\right)\Big]_0^{3/2}$$

$$= \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2u}{\sqrt{3}}\right)\Big]_0^{3/2}$$

$$= \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{3}{\sqrt{3}}\right) - 0$$

$$= \frac{2}{\sqrt{3}}\tan^{-1}(\sqrt{3})$$

$$= \frac{2}{\sqrt{3}}\cdot\frac{\pi}{3}$$

$$= \frac{2\pi}{3\sqrt{3}}$$

17. The improper integral $\int_0^{\pi/2} \frac{1}{\sqrt{1-\sin^2 x}}\,dx$ $= \lim_{t\to\frac{\pi}{2}^-} \int_0^t \frac{1}{\sqrt{1-\sin^2 x}}\,dx$

$= \lim_{t\to\frac{\pi}{2}^-} \int_0^t \sec x\,dx \quad , \quad 0 < t < \frac{\pi}{2}$

$= \lim_{t\to\frac{\pi}{2}^-} \ln|\sec x + \tan x|\Big]_0^t$

$= \lim_{t\to\frac{\pi}{2}^-} \ln|\sec t + \tan t| - 0$

$= \infty \quad \left(t \to \frac{\pi}{2}^- \Rightarrow \sec t \to \infty \;\&\; \tan t \to \infty\right)$

Div.

(a) diverges

(b) converges $\ln 2$

(c) converges to 1

(d) converges to 0

(e) converges to $\ln 3$

18. $\int_0^{\pi/2} \sin^3 x \cos(2x)\,dx =$ $\int_0^{\pi/2} \sin^2 x \cdot \cos(2x) \cdot \sin x\,dx$

$= \int_0^{\pi/2} (1-\cos^2 x)(2\cos^2 x - 1) \cdot \sin x\,dx$

$u = \cos x \Rightarrow du = -\sin x\,dx$

$x = 0 \Rightarrow u = 1$

$x = \frac{\pi}{2} \Rightarrow u = 0$

$= -\int_1^0 (1-u^2)(2u^2-1)\,du$

$= \int_0^1 2u^2 - 1 - 2u^4 + u^2\,du$

$= \int_0^1 3u^2 - 1 - 2u^4\,du$

$= u^3 - u - \frac{2}{5}u^5\Big]_0^1$

$= \left(1 - 1 - \frac{2}{5}\right) - 0$

$= -\frac{2}{5}$

(a) $-\frac{2}{5}$

(b) $\frac{8}{5}$

(c) 1

(d) 0

(e) $\frac{4}{5}$

$x = \tan t \Rightarrow dx = \sec^2 t \, dt$

19. $\displaystyle\int_{\pi/6}^{\pi/4} \frac{\sec^2 t}{\tan^2 t - 2\tan t}\, dt =$

$$\int_{1/\sqrt{3}}^{1} \frac{1}{x^2-2x}\, dx = \int_{1/\sqrt{3}}^{1} \frac{-1/2}{x} + \frac{1/2}{x-2}\, dx$$

$$= \frac{1}{2} \cdot -\ln|x| + \ln|x-2| \Big]_{1/\sqrt{3}}^{1}$$

$$= \frac{1}{2} \cdot \ln\left|\frac{x-2}{x}\right| \Big]_{1/\sqrt{3}}^{1}$$

$$= \frac{1}{2} \cdot \left(0 - \ln\left|\frac{1/\sqrt{3}-2}{1/\sqrt{3}}\right|\right)$$

$$= -\frac{1}{2}\ln\left|1-2\sqrt{3}\right|$$

$$= -\frac{1}{2}\ln(2\sqrt{3}-1)$$

$$= -\ln\sqrt{2\sqrt{3}-1}$$

(a) $-\ln\sqrt{2\sqrt{3}-1}$

(b) $\ln\left(\frac{2}{\sqrt{3}} - 1\right)$

(c) 0

(d) $1 - \ln(\sqrt{3}-1)$

(e) $\ln(2-\sqrt{3})$

$u = \sqrt[3]{x} \Rightarrow x = u^3 \Rightarrow dx = 3u^2\, du$

20. $\displaystyle\int_1^8 \frac{4x}{x + \sqrt[3]{x}}\, dx =$

$$\int_1^2 \frac{4u^3}{u^3+u} \cdot 3u^2\, du = 12\int_1^2 \frac{u^5}{u^3+u}\, du$$

$$= 12\int_1^2 \frac{u^4}{u^2+1}\, du$$

$$= 12\int_1^2 u^2 - 1 + \frac{1}{u^2+1}\, du$$

$$= 12 \cdot \left[\frac{1}{3}u^3 - u + \tan^{-1} u\right]_1^2$$

$$= 12 \cdot \left[\left(\frac{8}{3} - 2 + \tan^{-1} 2\right) - \left(\frac{1}{3} - 1 + \tan^{-1} 1\right)\right]$$

$$= 12 \cdot \left[\frac{7}{3} - 1 + \tan^{-1} 2 - \frac{\pi}{4}\right]$$

$$= 12 \cdot \left[\frac{4}{3} + \tan^{-1} 2 - \frac{\pi}{4}\right]$$

$$= 16 + 12\tan^{-1} 2 - 3\pi$$

$$= 16 - 3\pi + 12\tan^{-1} 2$$

(a) $16 - 3\pi + 12\tan^{-1} 2$

(b) $\pi + \ln(1 + \tan^{-1} 2)$

(c) $16 - \pi - 6\tan^{-1} 2$

(d) $4 - \frac{\pi}{4} + \tan^{-1} 2$

(e) $12 - 2\pi - 3\tan^{-1} 2$