Quiz 2

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King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 102 - 25 Quiz I (Term 162)

Name : ....KEY........ **ID #**.................................... Serial #: ....

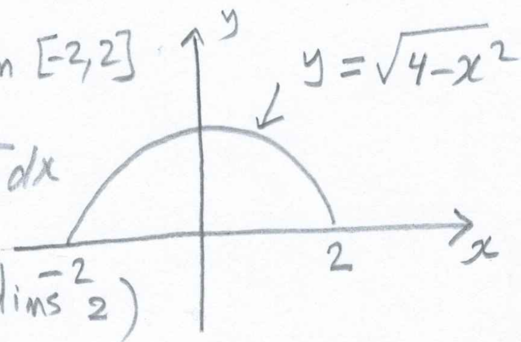
1. (4 points) Evaluate $\int_{-2}^{2} \sqrt{16-4x^2}\,dx$ by expressing it in terms of area.

$y = \sqrt{16-4x^2} = 2\sqrt{4-x^2}$ on $[-2,2]$

$\Rightarrow \int_{-2}^{2} \sqrt{16-4x^2}\,dx = 2\int_{-2}^{2}\sqrt{4-x^2}\,dx$

$= \cancel{2}\left(\frac{1}{\cancel{2}}\text{ area of circle of radius } 2\right)$

$= 4\pi.$



2. (5 points) Without the use of areas, evaluate $\int_1^0 |2x-1|\,dx$ $= -\int_0^1 |2x-1|\,dx$

$|2x-1| = \begin{cases} -2x+1, & 0 \le x \le \frac{1}{2} \\ 2x-1, & \frac{1}{2} < x \le 1 \end{cases} \Rightarrow$

$-\int_0^1 |2x-1|\,dx = -\int_0^{1/2} (-2x+1)\,dx - \int_{1/2}^{1} (2x-1)\,dx$

$= -\left[(-x^2+x)\right]_0^{1/2} - \left[x^2-x\right]_{1/2}^{1}$

$= -\left[\left(-\frac{1}{4}+\frac{1}{2}\right)-0\right] - \left[0-\left(\frac{1}{4}-\frac{1}{2}\right)\right]$

$= -\frac{1}{4} - \frac{1}{4} = -\frac{1}{2}.$

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3. (5 points) If $F(x) = \int_{\cos 2x}^{\tan x} \ln(1+2t)\, dt$ then find the value of $F'\left(\frac{\pi}{4}\right)$.

$$F'(x) = [\ln(1+2\tan x)](\sec^2 x) - [\ln(1+2\cos 2x)](-2\sin 2x)$$

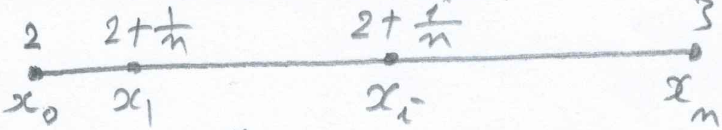
$$F'\left(\frac{\pi}{4}\right) = (\ln 3)(2) - [\ln 1](-2)$$

$$= 2\ln 3$$

OR $= \ln 9$

4. (6 points) Express the integral $\int_2^3 (x^2-4)\,dx$ as a limit of a Riemann Sum, then evaluate the limit. **[No other method will be accepted]**

$$\Delta x = \frac{3-2}{n} = \frac{1}{n}$$



$$\int_2^3 (x^2-4)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(2+\frac{i}{n}\right)^2 - 4\right]\left(\frac{1}{n}\right)$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\cancel{4} + \frac{4i}{n} + \frac{i^2}{n^2} - \cancel{4}\right] \frac{1}{n}$$

$$= \lim_{n\to\infty} \left[\frac{4}{n^2} \sum_{i=1}^{n} i + \frac{1}{n^3} \sum_{i=1}^{n} i^2\right]$$

$$= \lim_{n\to\infty} \left[\frac{4}{n^2} \frac{n(n+1)}{2} + \frac{1}{n^3} \frac{n(n+1)(2n+1)}{6}\right]$$

$$= 2 + \frac{1}{3} = \frac{7}{3}.$$