KFUPM Term 162 NAME: Solutions Serial#-------

MATH 102-25 Quiz 4 ID#------------------

1)(4 points) Find $\int \sin 5x . \sin 7x \, dx$

$$\sin\alpha \sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right] \Rightarrow$$

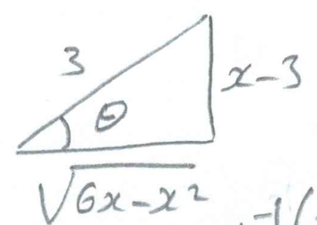
$$\int \sin 5x . \sin 7x \, dx = \frac{1}{2}\int\left[\cos(2x) - \cos(12x)\right] dx$$

$$= \frac{1}{4}\sin(2x) - \frac{1}{24}\sin(12x) + C$$

2)(6 points) Find $\int \frac{3x-12}{\sqrt{6x-x^2}} dx$ $= \int \frac{3x-12}{\sqrt{9-(x-3)^2}} dx = I$

Let $x-3 = 3\sin\theta \Rightarrow 9-(x-3)^2 = 9\cos^2\theta$,

$dx = 3\cos\theta \, d\theta$ and $x = 3+3\sin\theta$



$\theta = \sin^{-1}\left(\frac{x-3}{3}\right)$

$$\Rightarrow I = \int \frac{9+9\sin\theta-12}{3\cancel{\cos\theta}} 3\cancel{\cos\theta} \, d\theta$$

$$= \int (9\sin\theta - 3) \, d\theta = -9\cos\theta - 3\theta + C$$

$$= -9\frac{\sqrt{6x-x^2}}{3} - 3\sin^{-1}\left(\frac{x-3}{3}\right) + C$$

$$= -3\sqrt{6x-x^2} - 3\sin^{-1}\left(\frac{x-3}{3}\right) + C.$$

3)(4 points) Find $\int (tan4x)^3 \sqrt{sec4x}\, dx$ $= I$

$$I = \int (\sec^2 4x - 1)(\sec 4x)^{-1/2}(\tan 4x \sec 4x)\, dx$$

Let $u = \sec 4x \Rightarrow du = 4 \sec 4x \tan 4x\, dx \Rightarrow$

$$I = \frac{1}{4}\int \left(u^{3/2} - u^{-1/2}\right) du = \frac{1}{4}\left[\frac{u^{5/2}}{5/2} - \frac{u^{1/2}}{1/2}\right] + C$$

$$= \frac{1}{10}(\sec 4x)^{5/2} - \frac{1}{2}(\sec 4x)^{1/2} + C$$

4)(6 points) Represent the integral $\int \frac{7x^2-x+9}{(x-1)(x^2+4)} dx$ as a sum of integrals of the partial fractions of the integrand .**[Evaluate the constants BUT do not evaluate the integrals]**

$$\frac{7x^2-x+9}{(x-1)(x^2+4)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+4}$$

$$\Rightarrow 7x^2 - x + 9 = A(x^2+4) + (Bx+C)(x-1)$$

$\boxed{x=1} \Rightarrow 15 = 5A \Rightarrow \boxed{A=3}$

$\boxed{\text{Coef } x^2} \Rightarrow 7 = A + B \Rightarrow \boxed{B=4}$

$\boxed{\text{Const}} \Rightarrow 9 = 4A - C \Rightarrow C = 4A - 9 = \boxed{3 = C}$

$$\Rightarrow \int \frac{7x^2-x+9}{(x-1)(x^2+4)} dx = \int \frac{3}{x-1} dx + \int \frac{4x+3}{x^2-4} dx$$