KFUPM Term 162 NAME: ------------KEY------------ Serial#-------

MATH 102-25 Quiz 6 ID#-------------------

1)In (A) and (B) Test the series for convergence or divergence. If it is convergent, find its sum.

(A)(5 *points*) $\sum_{n=3}^{n=\infty} \frac{\sqrt{2n+1}-\sqrt{2n-1}}{\sqrt{4n^2-1}}$

$$= \sum_{n=3}^{\infty} \frac{\sqrt{2n+1}-\sqrt{2n-1}}{\sqrt{2n-1}\,\sqrt{2n+1}}$$

$$= \sum_{n=3}^{\infty} \left(\frac{1}{\sqrt{2n-1}} - \frac{1}{\sqrt{2n+1}}\right) \Rightarrow$$

$$S_n = \left(\frac{1}{\sqrt{5}} - \cancel{\frac{1}{\sqrt{7}}}\right) + \left(\cancel{\frac{1}{\sqrt{7}}} - \cancel{\frac{1}{\sqrt{9}}}\right) + \cdots + \left(\cancel{\frac{1}{\sqrt{2n-1}}} - \frac{1}{\sqrt{2n+1}}\right) \Rightarrow$$

$$S_n = \frac{1}{\sqrt{5}} - \frac{1}{\sqrt{2n+1}}$$

$$\lim_{n\to\infty} S_n = \frac{1}{\sqrt{5}}$$

$\Rightarrow$ The series is convergent and has the sum $\frac{1}{\sqrt{5}}$.

2)(5 points) $\sum_{n=1}^{n=\infty} (-1)^{n-1} \frac{2^{3n+1}}{11^{n+1}}$

$$= \sum_{n=1}^{\infty} (-1)^{n-1} \left(\frac{2}{11}\right)\left(\frac{2^{3n}}{11^n}\right)$$

$$= \sum_{n=1}^{\infty} (-1)^{n-1} \left(\frac{2}{11}\right)\left(\frac{8}{11}\right)^n$$

which is a geometric series with

Common ratio $= -\frac{8}{11}$

$\left|-\frac{8}{11}\right| < 1 \Rightarrow$

The series is convergent and has the sum

$$= \frac{\text{The first Term}}{1 - \text{Common Ratio}}$$

$$= \frac{\left(\frac{2}{11}\right)\left(\frac{8}{11}\right)}{1-\left(-\frac{8}{11}\right)} = \frac{16}{11^2+88}$$

$$= \frac{16}{121+88} = \frac{16}{209}.$$

3)(5 points) Verify that the integral test is applicable to the following series. Then use it to test the series for convergence or divergence.

$\sum_{n=1}^{\infty} \frac{1}{n^2+4n+5}$

Let $f(x) = \frac{1}{x^2+4x+5}$, $x \geq 1$

$\Rightarrow$ f is positive and continuous for all $x \geq 1$

$f'(x) = \frac{-(2x+4)}{(x^2+4x+5)^2} < 0 \Rightarrow$

f is decreasing for all $x \geq 1$

$\Rightarrow$ the Integral test is applicable

$\int_1^{\infty} \frac{1}{x^2+4x+5}\,dx$

$= \lim_{t\to\infty} \int_1^t \frac{1}{1+(x+2)^2}\,dx$

$= \lim_{t\to\infty} \tan^{-1}(x+2)\Big]_1^t$

$= \lim_{t\to\infty} \tan^{-1}(t+2) - \tan^{-1} 3$

$= \frac{\pi}{2} - \tan^{-1} 3 \Rightarrow$ The series is convergent.

4)(5 points) Use the direct or the limit comparison test to determine whether the following series is convergent or divergence'

$\sum_{n=1}^{\infty} \frac{n^2+n+3}{\sqrt[3]{n^7+5n+8}}$

$a_n = \frac{n^2+n+3}{\sqrt[3]{n^7+5n+8}}$

Compare with $b_n = \frac{n^2}{n^{7/3}}$

$\Rightarrow b_n = \frac{1}{n^{1/3}}$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n^{7/3}+n^{4/3}+3n^{1/3}}{(n^7+5n+8)^{1/3}}$

$= 1 > 0$

But $\sum \frac{1}{n^{1/3}}$ is a divergent p-series ($p = \frac{1}{3} < 1$)

$\Rightarrow$ The given series is divergent by the limit Comparison test.