Quiz 1 Page ①

King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 102 - 12 Quiz I (Term 162)

Name : ....KEY........................ **ID #**.................................... Serial #: ....

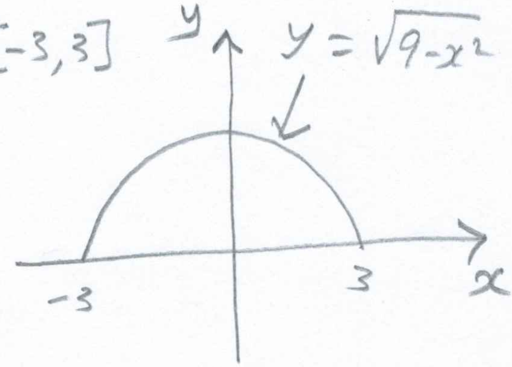
1. (4 points) Evaluate $\int_{-3}^{3} \sqrt{36-4x^2}\,dx$ by expressing it in terms of area.

$y = \sqrt{36-4x^2} = 2\sqrt{9-x^2}$ on $[-3,3]$

$\Rightarrow \int_{-3}^{3} \sqrt{36-4x^2} = 2\int_{-3}^{3} \sqrt{9-x^2}\,dx$

$= 2\left(\frac{1}{2}\text{ area of circle of radius } 3\right)$

$= 9\pi.$



2. (5 points) Without the use of areas, evaluate $\int_{-1}^{0} |2x+1|\,dx$

$$|2x+1| = \begin{cases} -2x-1, & -1 \le x \le -\frac{1}{2} \\ 2x+1, & -\frac{1}{2} < x \le 0 \end{cases}$$

$$\Rightarrow \int_{-1}^{0} |2x+1|\,dx = \int_{-1}^{-1/2} (-2x-1)\,dx + \int_{-1/2}^{0} (2x+1)\,dx$$

$$= \left[-x^2-x\right]_{-1}^{-1/2} + \left[x^2+x\right]_{-\frac{1}{2}}^{0}$$

$$= \left[\left(-\frac{1}{4}+\frac{1}{2}\right)-(-1+1)\right] + \left[0-\left(\frac{1}{4}-\frac{1}{2}\right)\right]$$

$$= \frac{1}{4}+\frac{1}{4} = \frac{1}{2}.$$

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Math 102 - 12 **Quiz I (Term 162)**

3. (5 points) If $F(x) = \int_{\sin 2x}^{\cot x} \ln(1+2t)\,dt$ then find the value of $F'\left(\frac{\pi}{4}\right)$.

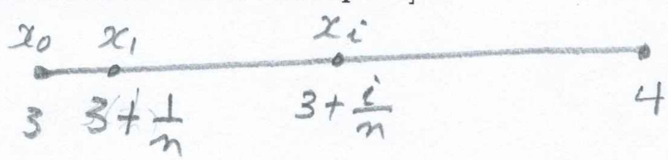
$$F'(x) = \left[\ln(1+2\cot x)\right](-\csc^2 x)$$
$$- \left[\ln(1+2\sin 2x)\right](4\cos 2x)$$

$$F'\left(\frac{\pi}{4}\right) = (\ln(3))(-2) - 0 = 2\ln 3$$

OR $= \ln 9.$

4. (6 points) Express the integral $\int_3^4 (x^2-9)\,dx$ as a limit of a Riemann Sum, then evaluate the limit. **[No other method will be accepted]**

$$\Delta x = \frac{4-3}{n} = \frac{1}{n}$$



$$\int_3^4 (x^2-9)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(3+\frac{i}{n}\right)^2 - 9\right]\left(\frac{1}{n}\right)$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\cancel{9} + \frac{6i}{n} + \frac{i^2}{n^2} - \cancel{9}\right]\frac{1}{n}$$

$$= \lim_{n\to\infty} \left[\frac{6}{n}\sum_{i=1}^{n} i + \frac{1}{n^3}\sum_{i=1}^{n} i^2\right]$$

$$= \lim_{n\to\infty} \left[\frac{6}{n}\,\frac{n(n+1)}{2} + \frac{1}{n^3}\,\frac{(n)(n+1)(2n+1)}{6}\right]$$

$$= 3 + \frac{1}{3} = \frac{10}{3}.$$