KFUPM Term 162 NAME: Solutions ---------- Serial#-------

MATH 102-12 Quiz 4 ID#----------

1)(4 points) Find $\int \cos 4x . \cos 6x \, dx$

$$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right] \Rightarrow$$

$$\int \cos 4x \cdot \cos 6x \, dx = \frac{1}{2}\int (\cos 2x + \cos 10x)\, dx$$

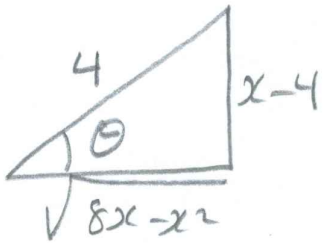
$$= \frac{1}{4}\sin 2x + \frac{1}{20}\sin 10x + C$$

2)(6 points) Find $\int \frac{4x+1}{\sqrt{8x-x^2}}dx = \int \frac{4x+1}{\sqrt{16-(x-4)^2}}dx = I$

Let $x-4 = 4\sin\theta \Rightarrow 16-(x-4)^2 = 16\cos^2\theta$,

$dx = 4\cos\theta\, d\theta$, $x = 4+4\sin\theta$,

and $\theta = \sin^{-1}\left(\frac{x-4}{4}\right)$. $\Rightarrow$



$$I = \int \frac{16 + 16\sin\theta + 1}{4\cancel{\cos\theta}}\, 4\cancel{\cos\theta}\, d\theta$$

$$= \int (17 + 16\sin\theta)\, d\theta = 17\theta - 16\cos\theta + C$$

$$= 17\sin^{-1}\left(\frac{x-4}{4}\right) - 16\,\frac{\sqrt{8x-x^2}}{4} + C$$

$$= 17\sin^{-1}\left(\frac{x-4}{4}\right) - 4\sqrt{8x-x^2} + C.$$

3)(4 points) Find $\int (tan5x)^3 \, (sec5x)^{\frac{3}{2}} \, dx$ $= I$

$$I = \int (\sec^2 5x - 1)(\sec 5x)^{1/2} (\sec 5x \cdot \tan 5x \, dx)$$

Let $u = \sec 5x \Rightarrow du = 5 \sec 5x \cdot \tan 5x \, dx$

$$\Rightarrow I = \frac{1}{5} \int (u^{9/2} - u^{1/2}) \, du$$

$$= \frac{2}{35} u^{7/2} - \frac{2}{15} u^{3/2} + C$$

$$= \frac{2}{35} (\sec 5x)^{7/2} - \frac{2}{15} (\sec 5x)^{3/2} + C.$$

4)(6 points) Represent the integral $\int \frac{3x^2+3x+14}{(x+1)(x^2+6)} dx$ as a sum of integrals of the partial fractions of the integrand .[**Evaluate the constants BUT do not evaluate the integrals]**

$$\frac{3x^2+3x+14}{(x+1)(x^2+6)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+6} \Rightarrow$$

$$3x^2+3x+14 = A(x^2+6) + (Bx+C)(x+1)$$

$x=-1 \Rightarrow 14 = 7A \Rightarrow \boxed{A=2}$

Coef $x^2 \Rightarrow 3 = A+B \Rightarrow \boxed{B=1}$

Const $\Rightarrow 14 = 6A + C \Rightarrow \boxed{C=2}$

$\Rightarrow$

$$\int \frac{3x^2+3x+14}{(x+1)(x^2+6)} dx = \int \frac{2}{x+1} dx + \int \frac{x+2}{x^2+6} dx.$$