

Name: _____

ID number: _____

1.) (5pts) Solve the IVP: $\begin{cases} y \frac{dy}{dx} = (y-3)(y+1) \cos x \sin^3 x \\ y(0) = 1. \end{cases}$

2.) (5pts) Solve the DE: $(x+1) \frac{dy}{dx} + y = \frac{x+1}{x^2+1}$.

$$1.) \int \frac{y}{(y-3)(y+1)} dy = \int \cos x \sin^3 x dx$$

$$\wedge \\ = \frac{a}{y-3} + \frac{b}{y+1}$$

$$\Rightarrow y = a(y+1) + b(y-3)$$

$$y=1 \Rightarrow b=1/4$$

$$y=3 \Rightarrow a=3/4$$

$$\int \left(\frac{3/4}{y-3} + \frac{1/4}{y+1} \right) dy = \frac{\sin^4 x}{4} + C$$

$$\frac{3}{4} \ln|y-3| + \frac{1}{4} \ln|y+1| = \frac{\sin^4 x}{4} + C$$

$$\ln|(y-3)^3(y+1)| = \sin^4 x + C$$

$$(y-3)^3(y+1) = C e^{\sin^4 x}$$

$$y(0)=1 \Rightarrow -16 = C$$

$$(y-3)^3(y+1) = -16 e^{\sin^4 x}, \\ x \in (-\pi, \pi)$$

$$2.) \frac{dy}{dx} + \frac{1}{x+1} y = \frac{1}{x^2+1}$$

$$e^{\int \frac{dx}{x+1}} = e^{\ln|x+1|} = x+1, \quad x > -1$$

$$\frac{d}{dx} [y(x+1)] = \frac{x+1}{x^2+1}$$

$$y(x+1) = \int \frac{x+1}{x^2+1} dx$$

$$= \int \left(\frac{1}{2} \frac{2x}{x^2+1} + \frac{1}{x^2+1} \right) dx$$

$$= \frac{1}{2} \ln(x^2+1) + \tan^{-1} x + C$$

$$y = \frac{1}{x+1} \left(\frac{1}{2} \ln(x^2+1) + \tan^{-1} x + C \right), \\ x \in (-1, \infty)$$