MATH 202.5 (Term 162)

Quiz 3 (Sects. 4.2, 4.3 & 4.5) Duration: 20min

Name: ID number:

1.)(3pts)Use reduction of order to find a second solution y_2 of the DE: $(1-x^2)y''+2xy'-2y=0$, giving that $y_1=1+x^2$ is a solution.
2.)(3pts)Solve the DE: $2y'''-3y''-2y'+3y=0$.
3.)(4pts) Find an operator of lowest order that annihilates $f(x)=x^3e^{-x}(\sin^2 x+e^x)$.

1.) $y'' + \frac{2x}{1-x^2}y' - \frac{2}{1-x^2}y = 0$

$$y_2 = y_1 \int \frac{e^{-\int P(x)\,dx}}{y_1^2}\,dx$$

$$e^{-\int P(x)dx} = e^{\int \frac{-2x}{1-x^2}} = e^{\ln|1-x^2|}$$
$$= 1-x^2, \quad x \in (-1,1)$$

$$\Rightarrow y_2 = (1+x^2)\int \frac{1-x^2}{(1+x^2)^2}\,dx$$

$$\frac{1-x^2}{(1+x^2)^2} = \frac{1+x^2}{(1+x^2)^2} - \frac{2x^2}{(1+x^2)^2}$$
$$= \frac{1}{1+x^2} - \frac{2x^2}{(1+x^2)^2}$$

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1}x$$

$$-\int \frac{2x^2}{(1+x^2)^2}\,dx = \frac{x}{1+x^2} - \int \frac{1}{1+x^2}\,dx$$ Integration by parts

$$= \frac{x}{1+x^2} - \tan^{-1}x$$

$$\Rightarrow \quad y_2 = (1+x^2)\,\frac{x}{1+x^2} = \underline{\underline{x}}$$

2.) auxiliary equation:

$$2m^3 - 3m^2 - 2m + 3 = 0$$
$$m^2(2m-3) - (2m-3) = 0$$
$$(m^2-1)(2m-3) = 0, \quad m = \pm 1,\ m = \frac{3}{2}$$

$$y = c_1e^{-x} + c_2e^{x} + c_3e^{3/2x}.$$

3.) $$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$\Rightarrow f(x) = x^3e^{-x}\left(\frac{1-\cos 2x}{2}\right) + x^3$$

$$= \frac{1}{2}x^3e^{-x} - \frac{1}{2}x^3e^{-x}\cos 2x + x^3$$

$$(D+1)^4(x^3) = 0$$

$$(D^2+2D+5)^4(x^3e^{-x}\cos 2x) = 0$$

$$D^4x^3 = 0$$

$$\Rightarrow D^4(D+1)^4(D^2+2D+5)^4.(f(x)) = 0$$