

Name: _____

ID number: _____

- 1.) (5pts) Find 2 powers series solutions of the DE: $(x-3)y'' + y' - 2y = 0$.
- 2.) (2pts) Find the indicial roots of the DE $2x^2y'' + x(x^5+1)y' - (3+x^2)y = 0$ at $x=0$.
- 3.) (3pts) Find a relation of recurrence satisfied by c_n , where $y = \sum_{n=0}^{\infty} c_n x^{n+\frac{1}{3}}$ is solution of the DE $3xy'' + 2y' + y = 0$.

$$1.) y = \sum_{n=0}^{\infty} c_n x^n, \quad |x| < \infty$$

$$(x-3) \sum_{n=2}^{\infty} n(n-1)c_n x^{n-2} + \sum_{n=1}^{\infty} n c_n x^{n-1} - 2 \sum_{n=0}^{\infty} c_n x^n = 0$$

$$-6c_2 + 4c_1 - 2c_0 + \sum_{k=1}^{\infty} [-3(k+1)(k+2)c_{k+2} + (k+1)^2 c_{k+1} - 2c_k] x^k = 0$$

$$\begin{cases} c_2 = \frac{4c_1 - 2c_0}{6} \\ c_{k+2} = \frac{(k+1)^2 c_{k+1} - 2c_k}{3(k+1)(k+2)}, \quad k=1,2,3, \dots \end{cases}$$

Case 1: $C_0 = 0, C_1 \neq 0$

$$c_2 = \frac{c_1}{6}, \quad c_3 = -\frac{2}{27} c_1$$

$$y = c_1 \left(x + \frac{x^2}{6} - \frac{2}{27} x^3 + \dots \right)$$

Case 2: $C_0 \neq 0, C_1 = 0$

$$c_2 = -\frac{c_0}{3}, \quad c_3 = -\frac{2}{27} c_0, \quad c_4 = \dots$$

$$y = c_0 \left(1 - \frac{x^2}{3} - \frac{2}{27} x^3 + \dots \right)$$

$y = a_1 y_1 + a_2 y_2$ is the general solution

$$2.) p(x) = \frac{1}{2}(1+x^5), \quad q(x) = -\frac{1}{2}(3+x^2)$$

$$r(r-1) + \frac{r}{2} - \frac{3}{2} = 0$$

$$2r^2 - r - 3 = 0, \quad r = -1, \frac{3}{2}$$

$$3.) 3x \sum_{n=0}^{\infty} (n+\frac{1}{3})(n-\frac{2}{3}) c_n x^{n-\frac{5}{3}} + 2 \sum_{n=0}^{\infty} (n+\frac{1}{3}) c_n x^{n-\frac{2}{3}} + \sum_{n=0}^{\infty} c_n x^{\frac{1}{3}} = 0$$

$$\frac{1}{3} x \left[3 \sum_{n=0}^{\infty} (n+\frac{1}{3})(n-\frac{2}{3}) c_n x^{n-1} + 2 \sum_{n=0}^{\infty} (n+\frac{1}{3}) c_n x^{n-1} + \sum_{n=0}^{\infty} c_n x^n \right] = 0$$

$$\sum_{k=0}^{\infty} 3(k+\frac{4}{3})(k+\frac{1}{3}) c_{k+1} x^k + \sum_{k=0}^{\infty} 2(k+\frac{4}{3}) c_{k+1} x^k + \sum_{k=0}^{\infty} c_k x^k = 0$$

$$\sum_{k=0}^{\infty} [(3k+4)(k+1) c_{k+1} + c_k] x^k = 0$$

$$c_{k+1} = -\frac{c_k}{(k+1)(3k+4)}, \quad k=0,1,2, \dots$$