MATH 202.5 (Term 162)

Quiz 6 (Sects. 8.2 & 8.3) Duration: 30min

Name: **ID number:**

1.)(4pts)Solve the homogeneous linear system $X' = \begin{pmatrix} 6 & -1 \\ 1 & 4 \end{pmatrix} X.$

2.)(3pts)Solve the homogeneous linear system $X' = \begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix} X.$

3.)(3pts)Solve the system $X' = AX + \begin{pmatrix} 3 \\ e^t \end{pmatrix}$**, given that** $\Phi(t) = \begin{pmatrix} e^t & -2e^{-2t} \\ e^t & e^{-2t} \end{pmatrix}$ **is a fundamental matrix of** $X' = AX$.

1(a)) $\begin{vmatrix} 6-\lambda & -1 \\ 1 & 4-\lambda \end{vmatrix} = 0$, $(6-\lambda)(4-\lambda)+1=0$

$\lambda^2 - 10\lambda + 25 = 0$, $\lambda = 5;5$

$(A-5I)K = 0$

$\left(\begin{array}{cc|c} 1 & -1 & 0 \\ 1 & -1 & 0 \end{array}\right)$ $\left(\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right)$ $x - y = 0$ $K\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$X_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{5t}$

$X_2 = (tK + P)e^{5t}$, $(A-5I)P = K$

$\left(\begin{array}{cc|c} 1 & -1 & 1 \\ 1 & -1 & 1 \end{array}\right)$ $\left(\begin{array}{cc|c} 1 & -1 & 1 \\ 0 & 0 & 0 \end{array}\right)$ $x - y = 1$ $P\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$X_2 = \left[t\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right] e^{5t} = \begin{pmatrix} t+1 \\ t \end{pmatrix} e^{5t}$

$$\boxed{X = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{5t} + c_2 \begin{pmatrix} t+1 \\ t \end{pmatrix} e^{5t}}$$

2.) $\begin{vmatrix} 3-\lambda & 4 \\ -4 & 3-\lambda \end{vmatrix} = 0$, $(3-\lambda)^2 + 16 = 0$

$\lambda = 3 \pm 4i$

$(A - (3+4i)I)K = 0$

$\left(\begin{array}{cc|c} 3-3-4i & 4 & 0 \\ -4 & 3-3-4i & 0 \end{array}\right)$ $\left(\begin{array}{cc|c} -4i & 4 & 0 \\ -4 & -4i & 0 \end{array}\right)$

$\left(\begin{array}{cc|c} 1 & i & 0 \\ -i & 1 & 0 \end{array}\right)$ $\left(\begin{array}{cc|c} 1 & i & 0 \\ 0 & 0 & 0 \end{array}\right)$ $x + iy = 0$ $K\begin{pmatrix} i \\ -1 \end{pmatrix}$

$K = \underbrace{\begin{pmatrix} 0 \\ -1 \end{pmatrix}}_{B_1} + i\underbrace{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}_{B_2}$

$X_1 = \left[\begin{pmatrix} 0 \\ -1 \end{pmatrix}\cos 4t - \begin{pmatrix} 1 \\ 0 \end{pmatrix}\sin 4t\right] e^{3t} = \begin{pmatrix} -\sin 4t \\ -\cos 4t \end{pmatrix} e^{3t}$

$X_2 = \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix}\cos 4t + \begin{pmatrix} 0 \\ -1 \end{pmatrix}\sin 4t\right] e^{3t} = \begin{pmatrix} \cos 4t \\ -\sin 4t \end{pmatrix} e^{3t}$

$$\boxed{X = c_1 X_1 + c_2 X_2}$$

3.) $\Phi^{-1}(t) = \frac{1}{3e^{-t}} \begin{pmatrix} e^{-2t} & 2e^{-2t} \\ -e^{t} & e^{t} \end{pmatrix} = \frac{1}{3}\begin{pmatrix} e^{-t} & 2e^{-t} \\ -e^{2t} & e^{2t} \end{pmatrix}$

$\Phi^{-1}(t)F = \frac{1}{3}\begin{pmatrix} 3e^{-t} + 2e^{2t} \\ -3e^{2t} + e^{3t} \end{pmatrix}$

$\int \Phi^{-1} F = \frac{1}{3}\begin{pmatrix} -3e^{-t} + e^{2t} \\ -\frac{3}{2}e^{2t} + \frac{e^{3t}}{3} \end{pmatrix}$

$\underbrace{\Phi \int \Phi^{-1} F}_{X_p} = \frac{1}{3}\begin{pmatrix} 2e^{3t} - \frac{2}{3}e^{t} \\ -\frac{3}{2} + \frac{e^{t}}{3} - \frac{3}{2}e^{3t} + \frac{e^{4t}}{3} \end{pmatrix}$

$$\boxed{X = \Phi(t)C + X_p}$$