Exercise 1

Let

\[ f(x, y) = \begin{cases} 
  x^3 - y^3 & \text{if } (x, y) \neq (0, 0) \\
  \frac{x^3 - y^3}{x^2 + y^2} & \text{if } (x, y) = (0, 0) 
\end{cases} \]

Find \( \frac{\partial f}{\partial x}(0, 0) \) and \( \frac{\partial f}{\partial y}(0, 0) \). Show that \( f \) is not differentiable at \( (0, 0) \).
Exercise 2

Let $\Omega$ be an open bounded and convex set in $\mathbb{R}^n$ and $f : \overline{\Omega} \rightarrow \mathbb{R}^m$ be a $C^1(\overline{\Omega})$. Show that there exists $M > 0$ such that

$$||f(x) - f(y)|| \leq M||x - y||$$
Exercise 3

Find the critical points of \( f(x, y, z) = x^2 + y^2 + z^2 + 2xyz \) and determine their nature.
Exercise 4
Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by

$$f(x, y) = (x^2 - y^2, 2xy)$$

(a) Show that $f$ is one-to-one on $U = \{(x, y : x > 0)\}$.
(b) Find $V = f(U)$.
(c) Find $D_{f^{-1}}(0, 1)$.
Exercise 5
Show that the equations
\[ xy^2 + xz + yv^2 = 3 \]
\[ u^3yz + 2xv - u^2v^2 = 2 \]
have a unique solution \((u, v) = f(x, y) = (f_1(x, y), f_2(x, y))\) near the point 
\((1, 1, 1, 1, 1)\) and find \(Df(1, 1, 1)\)