King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
MATH411 - Advanced Calculus II
Exam II – Semester 162

Exercise 1

Let

$$f(x,y) = \begin{cases} \dfrac{x^3 - y^3}{x^2 + y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

Find $\frac{\partial f}{\partial x}(0,0)$ and $\frac{\partial f}{\partial y}(0,0)$. Show that f is not differentiable at $(0,0)$.

Exercise 2

Let Ω be an open bounded and convex set in $\mathbb{R}^n$ and $f : \overline{\Omega} \to \mathbb{R}^m$ be a $\mathcal{C}^1(\overline{\Omega})$. Show that there exists $M > 0$ such that

$$||f(x) - f(y)|| \leq M||x - y||$$

Exercise 3

Find the critical points of $f(x, y, z) = x^2 + y^2 + z^2 + 2xyz$ and determine their nature.

Exercise 4

Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by

$$f(x, y) = (x^2 - y^2, 2xy)$$

(a) Show that f is one-to-one on $U = \{(x, y : x > 0)\}$.

(b) Find $V = f(U)$.

(c) Find $D_{f^{-1}}(0, 1)$.

Exercise 5

Show that the equations

$$xy^2 + xzu + yv^2 = 3$$
$$u^3yz + 2xv - u^2v^2 = 2$$

have a unique solution $(u, v) = f(x, y) = (f_1(x, y), f_2(x, y)$ near the point $(1, 1, 1, 1, 1)$ and find $D_f(1, 1, 1)$