Problem 1 (14 points): Define each of the following

(a) Nonseparable graph

(b) Eccentricity

(c) Center of a graph

(d) Tournament

(e) The Caley tree theorem

(f) A caterpillar

(g) A block in a graph
Problem 2 (26 points): Consider the graph $G$ in the figure

(a) Find all cut-vertices (if any exists).

(b) Find all bridges of $G$ (if any exists).

(c) Draw all blocks of $G$.

(d) Find the center of $G$.

(e) Find the girth of $G$.

(f) Find the radius of $G$.

(g) Find the cycle rank of $G$.

(h) Without finding the adjacency matrix $M = \left[ m_{ij} \right]$ of $G$, find $m_{ij}^{(3)}$. 
Problem 3 (20 points): Either prove or disprove each of the following statements. If a statement is true sketch the proof, and if it is false, give a counter example.

1) If $G$ is a connected graph of order $n$ and size $m$ such that $m = n - 1$, then $G$ is a tree.

2) Every nontrivial connected graph has at least two vertices which are not cut vertices.
3) In a connected graph $G$, if every vertex has even degree, then the graph has no bridges.

4) If $G$ is a graph of order $n$ such that $\delta(G) \geq \frac{n-1}{2}$, then $G$ is connected.
Problem 4 (40 points):

(a) Construct the labeled tree having the Prufer code: (3,3,1,6,2,2).

(b) Show that the two graphs in the figure are not isomorphic.
(c) Prove that the score sequence $s_i$ of a tournament of order $n$ $(n \geq 3)$ satisfies the equation

$$\sum_{i=1}^{n} s_i^2 = \sum_{i=1}^{n} (n - 1 - s_i)^2.$$ 

(d) If $G_1$ and $G_2$ are regular graphs of degrees $r_1$ and $r_2$ respectively, then the Cartesian product $G_1 \times G_2$ is regular.