King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics Sciences
Math 425 - Graph Theory
Semester – 162

Exam II **Dr. M. Z. Abu-Sbeih** **April 18, 2017**

Student No.: __________. Name: ___________________________________

Show all your work. No credits for answers without justification.
Write neatly and eligibly. You may loose points for messy work.

Problem 1 (14 points): *Define each of the following*

(a) Nonseparable graph

(b) Eccentricity

(c) Center of a graph

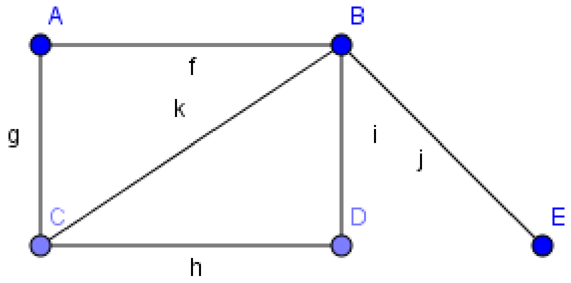
(d) Tournament

(e) The Caley tree theorem

(f) A caterpillar

(g) A block in a graph

Problem 2 (26 points): Consider the graph *G* in the figure



(a) Find all cut- vertices (if any exists).

(b) Find all bridges of *G* (if any exists).

(c) Draw all blocks of *G*.

(d) Find the center of *G*.

(e) Find the girth of *G*.

(f) Find the radius of *G*.

(g) Find the cycle rank of *G*.

(h) Without finding the adjacency matrix $M = \left[m_{ij} \right]$ of *G*, find $m_{ij}^{(3)}$.

Problem 3 (20 points): *Either prove or disprove each of the following statements. If a statement is true sketch the proof, and if it is false, give a counter example.*

1) If G is a connected graph of order n and size m such that $m = n - 1$, then G is a tree.

2) Every nontrivial connected graph has at least two vertices which are not cut vertices.

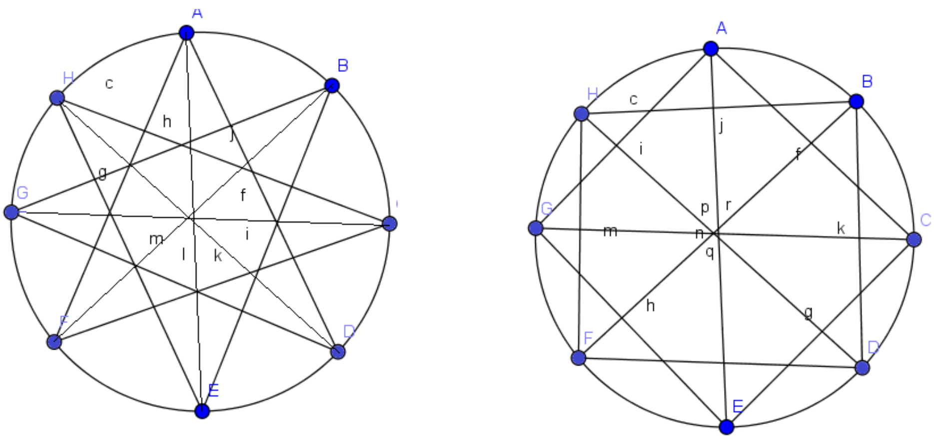
3) In a connected graph G, if every vertex has even degree, then the graph has no bridges.

4) If G is a graph of order n such that $\delta(G) \geq \frac{n-1}{2}$, then G is connected.

Problem 4 (40 points):

(a) Construct the labeled tree having the Prufer code: (3,3,1,6,2,2).

(b) Show that the two graphs in the figure are not isomorphic.



(c) Prove that the score sequence s_i of a tournament of order n $(n \geq 3)$ satisfies the equation

$$\sum_{i=1}^{n} s_i^2 = \sum_{i=1}^{n} (n-1-s_i)^2.$$

(d) If G_1 and G_2 are regular graphs of degrees r_1 and r_2 respectively, then the Cartesian product $G_1 \times G_2$ is regular.