King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics Sciences
Math 425 - Graph Theory
Duration: 8:00 – 11:00 AM

Final Exam **Dr. M. Z. Abu-Sbeih** **Wednesday May 24, 2017**

Student No.: ___________. Name: ____

Show all your work. No credits for answers without justification.
Write neatly and eligibly. You may lose points for messy work.

Make sure that you have 9 pages with 7 questions.

Page	Grade
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Total	

Problem 1 [16 marks]

(A) [8 marks] Define each of the following

(a) **A 1-tough graph**:

(b) **The line graph of a plane graph *G* of order *n* and size *m*:**

(c) **Perfect Matching:**

(d) **Edge cover:**

(B) [8 marks] State each of the following; define the terminology you use in the theorems:

a) **The Matrix-Tree Theorem.**

b) **Kutatowski's Theorem for planar graphs.**

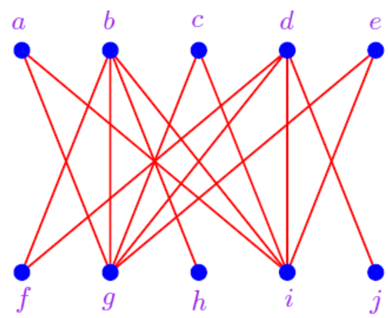
c) **Menger's Theorem.**

d) **Orthogonality relation between matrices of graphs.**

Problem 2 [14 marks]

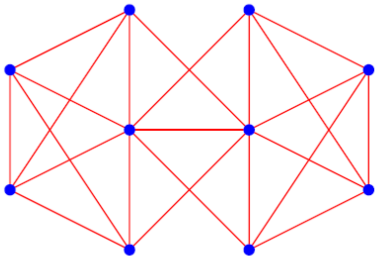
(A) [8 marks] Use Havel-Hakimi Theorem to determine whether the sequence is graphical or not. If yes draw the corresponding graph. (7,4,3,3,2,2,2,1)

(B) [6 marks] If exists, find a maximum matching and a minimum vertex cover in the following graph.

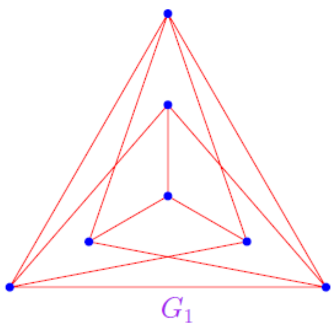


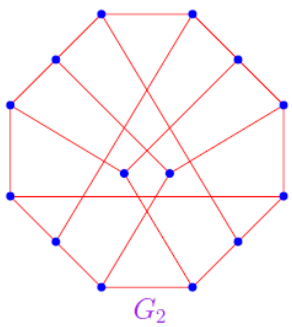
Problem 3 [18 marks]

(A) [6 marks] Determine the connectivity and the edge-connectivity of the graph from the picture.



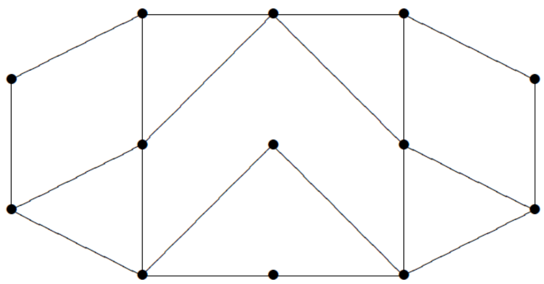
(B) [12 marks] For the graphs G_1 and G_2 from the picture, prove non-planarity or provide a planar embedding.



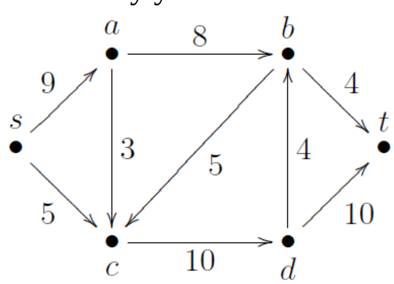


Problem 4 [18 marks]

(A) [6 marks]**:** Determine whether the given graph is Hamiltonian. If it is, find a Hamiltonian cycle. If it is not, prove it is not.



(B) [12 marks] Consider the network with source *s* and sink (terminal) *t*, and with the given capacity. Find a maximum flow. Justify your answer.



Problem 5 [22 marks]

(A) [12 marks] Let G be a graph of order $n > 4$ such that $d(v) \geq \frac{n-1}{2}$ for all vertices v of G. Prove that:

(a) G is connected.

(b) G contains a cycle.

(c) $diam(G) \leq 2$.

(d) G contains a Hamiltonian path.

(B) [10 marks] Answer each of the following. Sketch The graph if possible.

(a) The crossing number of $K_{1,2,3}$ is equal to: _____

(b) A maximal outer planar graph of order n must have size: __

(c) Find all connected graphs G where $G \cong L(G)$.

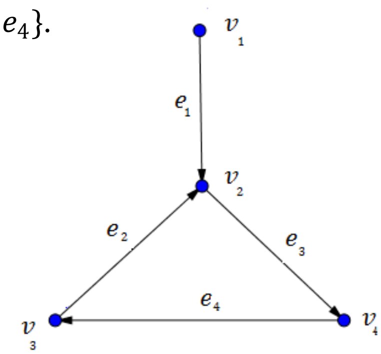
(d) Find a connected plane graph G which is isomorphic to its dual G^*

(e) If P_n is a path of order n $(n \geq 2)$, then $|Aut(P_n)| =$ ______

Problem 6 [20 marks]

(A) [10 marks] In a village there are three schools with n students in each of them. Every student from any of the schools is on speaking terms with at least $n + 1$ students from the other two schools. Show that we can find three students, no two from the same school, who are on speaking terms with each other.

(B) [10 marks] Consider the digraph G with the spanning tree $T = \{e_1, e_3, e_4\}$.



(a) Find The fundamental cutset matrix Q_f with respect to T

(b) Find The fundamental circuit matrix B_f with respect to T

(c) Arrange the columns of both Q_f and B_f in the same edge order and calculate $B_f Q_f^t$.

Problem 7 [32 marks] For each of the following statements decide if it is true or false. Give a succinct explanation.

1. Every 3-regular graph has a perfect matching.

2. There exists a 6-connected planar graph.

3. Every connected graph of order *n* and size $n-1$ is a tree.

4. The complete graph K_{2n+1} can be factored into Hamiltonian paths.

5. If a graph G has exactly two vertices u and v of odd degree, then G has a $u - v$ path.

6. If v is a cut vertex of a connected graph G, then v is a cut vertex of the complement $\bar{G}$.

7. Any cutest and any cycle of a graph have an even number of edges in common.

8. Every tournament contains a Hamiltonian path.