King Fahd University of Petroleum & Minerals  
Department of Mathematics & Statistics  
Math 430  Major Exam II  
The Second Semester of 2016-2017 (162)  
Time Allowed: 90 Minutes

Name: ______________________  ID#: ______________________
Section: ____________________  Serial #: ____________________

- Mobiles and calculators are not allowed in this exam.
- Provide all necessary steps required in the solution.

<table>
<thead>
<tr>
<th>Question #</th>
<th>Marks</th>
<th>Maximum Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>18</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>15</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>51</td>
</tr>
</tbody>
</table>
Q1: (3 + 6 + 9 points) (a) Define harmonic function for a real valued function of two variables.

(b) Verify that the function \( u(x, y) = x y^3 - x^3 y \) is harmonic in the entire complex plane. Find a harmonic conjugate of \( u(x, y) \).

(c) State and prove reflection principle.

Q2: (6 + 6 + 6 points) (a) (i) Find all values of \( (1 + i)^{(1-1)} \).

(ii) Solve the equation \( \cos z = \sqrt{2} \).

(b) Find all roots of the equation \( \cos(z) = \cosh 2 \) by equating the real and imaginary parts of \( \cos(z) \) and \( \cosh 2 \).

(c) Find all roots of the equation \( \sinh(z) = i \).

Q3: (2 + 5 + 8 points) (a) Show that if \( w(t) = u(t) + i v(t) \) is continuous on \( a \leq t \leq b \), then

\[
\int_a^b w(t) \, dt = \int_\alpha^\beta w[\phi(\tau)] \, \phi'(\tau) d\tau,
\]

where \( \phi(\tau) = t, \alpha \leq \tau \leq \beta \).

(b) Find the upper bound of \( \oint_C \frac{z + 4}{z^3 - 1} \, dz \), where \( C \) is the circle \(|z| = 2\) from \( z = 2 \) to \( z = 2i \).

(c) State the Cauchy-Goursat theorem and use it to evaluate \( \oint_C \frac{z + 1}{z^2 + 2z - 3} \, dz \), where \( C \) is the circle \(|z - 2| = 2\).