King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 430  Exam 03
The Second Semester of 2016-2017 (162)

Name: ___________________________  ID#: __________________
Section/Instructor: _________________  Serial #: _________________

- Mobiles and calculators are not allowed in this exam.
- Provide all necessary steps required in the solution.

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<th>Question #</th>
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Q1: (3 + 8 points) (a) State and prove Gauss’s mean value theorem.

(b) Use Cauchy’s integral formula(s) to evaluate

\[ \int_C \frac{z^3 + 3}{z(z - i)^2} dz, \]

where \( C \) is the contour shown in the fig.

Q2: (6 points) Let \( C \) be the unit circle \( z = e^{i\theta} \) \((-\pi \leq \theta \leq \pi)\). First show that for any real constant \( a \), \( \int_C \frac{e^{az}}{z} dz = 2\pi i \). Then write this integral in terms of \( \theta \) to derive the integral formula

\[ \int_0^\pi e^{a \cos(\theta)} \cos(a \sin \theta) d\theta = \pi. \]

Q3: (8 points) State and prove the fundamental theorem of algebra for any polynomial \( P(z) \) of degree \( n(n \geq 1) \).

Q4: (4 + 4 + 4 + 4 points) Let \( f(z) = (z^2 - 3z + 2)^{-1} \). Find the Laurent series for \( f(z) \) valid for

(a) \( 1 < |z| < 2 \) \hspace{1cm} (b) \( |z| < 1 \) \hspace{1cm} (c) \( |z| > 2 \) \hspace{1cm} (d) \( 0 < |z - 1| < 1 \)

Q5: (5 + 5 points) (a) Expand \( f(z) = \frac{1}{1 - z} \) in a Taylor series with center \( z_0 = 2i \) and find the circle of convergence.

(b) Evaluate

\[ \int_C \frac{1}{z^2 \sinh(z)} dz, \]

where \( C \) is the positively oriented circle \( |z| = 1 \) and

\[ \frac{1}{\sinh(z)} = \frac{1}{z} - \frac{1}{6}z^3 + \frac{7}{360}z^5 + \ldots \quad (0 < |z| < \pi). \]