Q 1.(a) If \( f^*(\alpha) \) denotes the Fourier transform of \( f(x) \) then show the duality property:

\[
F\left\{ f^*(x) \right\} = 2\pi f(-\alpha)
\]

(b) Find \( F\{e^{-x}\} \) and use duality property to find the Fourier transform of \( \frac{1}{x^2 + 1} \)

Q 2. Use the Fourier transform find the solution of the integral equation

\[
\phi(x) - \lambda \int_{-\infty}^{\infty} e^{-|x-y|} \phi(y) dy = f(x).
\]

Show that for \( f(x) = e^{-|x|} \), the solution can be written as \( \phi(x) = \frac{e^{-\sqrt{1-2\lambda}|x|}}{\sqrt{1-2\lambda}}, \lambda < \frac{1}{2} \).

Q 3. Solve the initial boundary value problem for heat equation:

\[
\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad -\infty < x < \infty, \quad t > 0
\]

\[
u(x,0) = g(x),
\]

\[
u(x,t), u_t(x,t) \to 0 \text{ as } |x| \to \infty
\]

Q 4. Solve using the Hankel transform
\[
\left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \right) u(r, z) = 0, \quad 0 \leq r < \infty, \quad z > 0
\]

\[
u(r, 0) = f(r), \quad 0 \leq r < \infty
\]

\[
\frac{\partial u(r, 0)}{\partial z} = 0, \quad u(r, z) \to 0 \quad \text{as } r \to \infty
\]