KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

Department of Mathematics & Statistics

Math 514                  Advanced Methods of Applied Mathematics

Final Exam                                                       Term 162

Time Allowed 2  Hours

Name__________________                              ID # _______

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<th>Q #</th>
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Important Note

Write clearly and show all work.

Instructor: F. D. Zaman
Q1) Solve the following integral equation using the *Wiener Hopf Technique*

\[ \int_0^\infty e^{-|x-\xi|} u(\xi) d\xi = -\frac{1}{4} u(x) + 1, \quad 0 < x < \infty. \]
Q2) Solve using the *Mellin transform*

\[ x^2 u_{xx} + xu_x + u_{yy} = 0, \ 0 \leq x < \infty, \ 0 < y < 1. \]

\[ u(x,0) = 0 \]

\[ u(x,1) = \begin{cases} 
1, & 0 \leq x \leq 1 \\
0, & x > 1.
\end{cases} \]
Q 3) Use the method of integration by parts to obtain asymptotic estimate of the following

\[ \text{Erfc}(x) = \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} e^{-t^2} dt. \]
Q4) Use *procedure* of the *Laplace method* to obtain an asymptotic estimate of the integral

\[ f(x) = \int_{-\infty}^{\infty} \exp(-x \cos t) dt, \quad x \to \infty. \]
Q 5) Use *Watson’s Lemma* to find asymptotic representation of following parts for $x \to \infty$

\[
(Watson’s~Lemma: \quad \int_0^T e^{-x^t} g(t) \, dt \approx \sum_{n=0}^{\infty} \frac{g^n(0) \Gamma(\lambda + n + 1)}{n! x^{\lambda + n + 1}}, \quad x \to \infty)
\]

\[
(a) \int_0^x e^{-x \sinh t} \, dt
\]
(b) $\int_{0}^{\pi/2} \sqrt{\sin t} \ e^{-x\sin^4 t} \ dt$