Department of Mathematics and Statistics, KFUPM

Midterm Exam for MATH 572, 27 April 2017

Problem 1. Consider the IVP: $y' = y^{1/5}$ with $y(0) = 0$.

a) Show that this IVP has at least two continuously differentiable solutions. Does this contradict Picard 's theorem? Justify your answer.

b) Compute both the explicit and implicit Euler solutions of the above IVP. Explain why the Euler solutions fail to approximate the non-zero solution of the IVP.

Problem 2. Consider the trapezium rule method

$$y^{n+1} - y^n = \frac{h}{2}(f^{n+1} + f^n), \quad y^0 = y_0,$$

for the numerical solution of the IVP: $y' = f(x,y)$ with $y(0) = y_0$ given, where $f^n = f(x_n, y^n)$ and $h = x_n - x_{n-1}$.

a) Show that the truncation error $T^n = \frac{1}{12}h^2 y'''(\xi_n)$ for some ξ_n in the interval (x_n, x_{n+1}).

b) Suppose that f satisfies the Lipschitz condition $|f(x,u) - f(x,v)| \leq L|u - v|$ where L is a positive constant, and that $|y'''(x)| \leq M$ for some positive constant M. Then the global error $e_n = y(x_n) - y^n$ satisfies the inequality

$$|e_{n+1}| \leq |e_n| + \frac{hL}{2}(|e_{n+1}| + |e_n|) + \frac{h^3}{12}M, \quad \text{for } n \geq 0.$$

Prove that

$$|e_n| \leq \frac{h^2 M}{12L}\left(\left(\frac{1 + hL/2}{1 - hL/2}\right)^n - 1\right) \quad \text{when } h < L/2.$$

Problem 3. Consider the following two-stage RK method for the numerical solution of the IVP: $y = f(x,y)$ with $y(x_0) = y_0$:

$$y^{n+1} = y^n + \frac{h}{2}(k_1 + k_2), \text{ where } k_1 = f^n \text{ and } k_2 = f(x_{n+1}, y^n + hk_1).$$

1- Verify that the method is consistent

2- Show that the truncation error is of order two.

3- Determine the interval of absolute stability

Problem 4. Use the rooted trees to determine the system of four equations for the six unknown parameters of the consistent third order accurate 3-stage RK method.