Q1. The UK retail sales from Jan 1986 to Mar 2007 was analyzed. Using the following summary and plot,

| Estimate  | Std. Error | t value | Pr(>|t|) |
|-----------|------------|---------|---------|
| Intercept | -7334.9763 | 307.0736| -23.89  | <2e-16 *** |
| time(retail) | 3.7171 | 0.1538 | 24.17  | <2e-16 *** |

Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 15.07 on 253 degrees of freedom
Multiple R-squared: 0.6978,  Adjusted R-squared: 0.6966
F-statistic: 584.1 on 1 and 253 DF,  p-value: < 2.2e-16

a) Write the time series model that was used in the analysis.
b) Describe any remaining patterns in the data that should be modeled.
c) Should you use the random cosine model for this data? Why or why not?

Q2. Suppose that a stationary time series \( \{ Y_t \} \), has an autocorrelation of \( \rho_k = 0.45^k \) for \( k > 0 \).

a) Compute \( \text{Var}(\overline{Y}) \)

(Hint: For \( |\lambda| < 1 \), \( \sum_{k=0}^{n} \lambda^k = \frac{1-\lambda^{n+1}}{1-\lambda} \) and \( \sum_{k=0}^{n} k\lambda^{k-1} = \frac{d}{d\lambda} [\sum_{k=0}^{n} \lambda^k] \))

b) For large \( n \), compare the precision of this series with the series \( Y_t = \mu + e_t \), where \( e_t \) is zero-mean white noise process.