King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 101 Section 7 Quiz II(B) (Term 163)

Name : ......KEY...................... **ID #**.................................... Serial #: ....

1. A particle moves according to a law of motion $s = f(t) = \sin\left(\frac{\pi t}{2}\right)$, $t \geq 0$, where t is measured in seconds and s in meters. When is the particle slowing down in the first 6 seconds?

$$V = f'(t) = \cos\left(\frac{\pi t}{2}\right) \cdot \frac{\pi}{2} = \frac{\pi}{2} \cos\left(\frac{\pi t}{2}\right)$$

$$a = v'(t) = f''(t) = \frac{-\pi^2}{4} \sin\left(\frac{\pi t}{2}\right)$$

$v(t)$		+ve		-ve		-ve		+ve		+ve		-ve	
	$t=0$		$t=1$		$t=2$		$t=3$		$t=4$		$t=5$		$t=6$
$\frac{\pi t}{2}$	0		$\frac{\pi}{2}$		π		$\frac{3\pi}{2}$		2π		$\frac{5\pi}{2}$		3π
$a(t)$		-ve		-ve		+ve		+ve		-ve		-ve	

The particle is slowing down if

$0 < t < 1$, $2 < t < 3$ or $4 < t < 5$.

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2. Find $f'(x)$ if $f(x) = (\sin x)^{\csc x}$

$\ln y = \csc x \ln(\sin x)$

$\Rightarrow \frac{y'}{y} = -\csc x \cot x \ln(\sin x) + \csc x * \frac{\cos x}{\sin x}$

$\Rightarrow y' = (\sin x)^{\csc x}\left[-\csc x \cot x \ln(\sin x) + \csc x \cot x\right]$

3. Find the instantaneous rate of change of $f(x) = \dfrac{x^2\, e^{\sqrt{x-1}}}{1-x}$ with respect to x at $x = 2$.

$y = \frac{x^2 e^{\sqrt{x-1}}}{1-x} \Rightarrow \ln y = 2\ln x + \sqrt{x-1} - \ln(1-x)$

$\Rightarrow \frac{y'}{y} = \frac{2}{x} + \frac{1}{2\sqrt{x-1}} + \frac{1}{1-x}$

$x=2 \Rightarrow y = \frac{4e}{-1} = -4e$

$\Rightarrow y'\Big|_{x=2} = -4e\left[1+\frac{1}{2}-1\right] = -2e$

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4. Find $\lim_{x\to\infty}\left(1-\frac{2}{5x}\right)^x$ $= \lim_{x\to\infty}\left(1+\frac{-2/5}{x}\right)^x = e^{-2/5}$

5. If $x^2+4y^2=4$ then find y'' in its simplest form.

$\Rightarrow 2x + 8yy' = 0 \Rightarrow y' = \frac{-2x}{8y} = \frac{-x}{4y}$

$\Rightarrow y'' = \frac{-4y+4xy'}{16y^2} = \frac{-4y - \frac{4x^2}{4y}}{16y^2} = \frac{-16y^2-4x^2}{64y^3}$

$= \frac{-4(x^2+4y^2)}{64y^3} = \frac{-1}{4y^3}$

6. Let $f(x)=5x+3e^{7x}$. Then find $\frac{df^{-1}}{dx}|_{x=3}$.

$\left.\frac{df^{-1}}{dx}\right|_{x=3} = \frac{1}{f'(f^{-1}(3))}$

To find $f^{-1}(3)$, we need to find x such that $f(x)=3$

$\Rightarrow 3 = 5x+3e^{7x} \Rightarrow x=0 \Rightarrow f^{-1}(3)=0$

Also, $f'(x) = 5+21e^{7x} \Rightarrow f'(0) = 5+21=26$

$\Rightarrow \left.\frac{df^{-1}}{dx}\right|_{x=3} = \frac{1}{26}$

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7. Evaluate $\lim_{x \to \frac{\pi}{4}} \dfrac{\cot x - 1}{x - \frac{\pi}{4}}$

If $y = f(x) = \cot x$, then $f'\left(\frac{\pi}{4}\right) = \lim_{x \to \frac{\pi}{4}} \dfrac{f(x) - f\left(\frac{\pi}{4}\right)}{x - \frac{\pi}{4}}$

$= \lim_{x \to \frac{\pi}{4}} \dfrac{\cot x - 1}{x - \frac{\pi}{4}}$

Note $f'(x) = -\csc^2 x \Rightarrow f'\left(\frac{\pi}{4}\right) = -2$

$\therefore \lim_{x \to \frac{\pi}{4}} \dfrac{\cot x - 1}{x - \frac{\pi}{4}} = f'\left(\frac{\pi}{4}\right) = -2$

8. For $t > 0$, find $\dfrac{d}{dt}\left[\sin^{-1}\left(\dfrac{t-4}{t+4}\right)\right]$ in its simplest form.

$\dfrac{d}{dt}\left[\sin^{-1}\left(\dfrac{t-4}{t+4}\right)\right] = \dfrac{1}{\sqrt{1-\left(\frac{t-4}{t+4}\right)^2}} * \left[\dfrac{t+4-t+4}{(t+4)^2}\right]$

$= \dfrac{t+4}{\sqrt{(t+4)^2 - (t-4)^2}} \left[\dfrac{8}{(t+4)^2}\right] = \dfrac{8}{(t+4)\sqrt{t^2+8t+16-t^2+8t-16}}$

$= \dfrac{8}{(t+4) * 4\sqrt{t}} = \dfrac{2}{\sqrt{t}(t+4)} = \dfrac{2\sqrt{t}}{t(t+4)}$