King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 101 Section 8 Quiz II(A) (Term 163)

Name : KEY **ID #**.................................... Serial #: ....

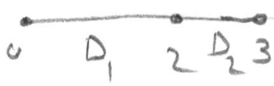
1. A particle moves according to a law of motion $s = f(t) = t^3 - 9t^2 + 24t$, $t > 0$, where t is measured in seconds and s in meters.

   (a) Find the total distance travelled during the first 3 seconds.

$V(t) = f'(t) = 3t^2 - 18t + 24 = 3(t^2 - 6t + 8)$

$= 3(t-2)(t-4)$

$D_1 = |f(2) - f(0)|$



$= |20 - 0| = 20$

$D_2 = |f(3) - f(2)| = |18 - 20| = 2$

$D = D_1 + D_2 = 20 + 2 = 22$ m

   (b) Find the acceleration when its velocity is equal to 24.

$V(t) = 24 \Rightarrow 3t^2 - 18t + 24 = 24 \Rightarrow 3t(t-6) = 0$

$\Rightarrow t = 6$ s

Now, $a(t) = 6t - 18$

$a(6) = 36 - 18 = 18$ m^2/s

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2. Find $f'(x)$ if $f(x) = (\ln x)^{\tan x}$

$y = (\ln x)^{\tan x} \Rightarrow \ln y = (\tan x)\ln(\ln x)$

Diff. w.r.t. $x \Rightarrow \frac{y'}{y} = \sec^2 x \ln(\ln x) + \frac{\tan x}{x \ln x}$

$\Rightarrow y' = (\ln x)^{\tan x}\left[\sec^2 x \ln(\ln x) + \frac{\tan x}{x \ln x}\right]$

3. If $y = \sqrt[3]{\frac{x(x+1)(x+2)}{(x+3)(x+4)(x+5)}}$, then find $y'|_{x=1}$.

$\ln y = \frac{1}{3}(\ln x + \ln(x+1) + \ln(x+2) - \ln(x+3) - \ln(x+4) - \ln(x+5))$

Diff. w.r.t. $x \Rightarrow$

$\frac{y'}{y} = \frac{1}{3}\left[\frac{1}{x} + \frac{1}{x+1} + \frac{1}{x+2} - \frac{1}{x+3} - \frac{1}{x+4} - \frac{1}{x+5}\right]$

At $x = 1 \Rightarrow y = \sqrt[3]{\frac{1(2)(3)}{4(5)(6)}} = \frac{1}{\sqrt[3]{20}}$

$\Rightarrow y'\Big|_{x=1} = \frac{1}{3\sqrt[3]{20}}\left[1 + \frac{1}{2} + \frac{1}{3} - \frac{1}{4} - \frac{1}{5} - \frac{1}{6}\right] = \frac{1}{3\sqrt[3]{20}}\left[\frac{60+30+20-15-12-10}{60}\right]$

$= \frac{73}{180\sqrt[3]{20}}$

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4. Find $\lim\limits_{x\to\infty}\left(1+\frac{5}{3x}\right)^x$ $= \lim\limits_{x\to\infty}\left(1+\frac{5/3}{x}\right)^x = e^{5/3}$

5. If $x^3 - y^3 = 7$, then find y'' in its simplest form.

Diff. w.r.t. x $\Rightarrow 3x^2 - 3y^2 y' = 0 \Rightarrow y' = \frac{x^2}{y^2}$

$\Rightarrow y'' = \frac{2xy^2 - 2x^2 y y'}{y^4} = \frac{2xy^2 - \frac{2x^4}{y}}{y^4} = \frac{2xy^3 - 2x^4}{y^5}$

$= \frac{2x(y^3 - x^3)}{y^5} = \frac{-14x}{y^5}$

6. Let $f(x) = 1 + 2x - x^2$, $x \le 1$. Then find $\frac{df^{-1}}{dx}|_{x=-2}$.

$\left.\frac{df^{-1}}{dx}\right|_{x=-2} = \frac{1}{f'(f^{-1}(-2))}$

To find $f^{-1}(-2)$, we need to find x such that

$-2 = 1 + 2x - x^2 \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow (x-3)(x+1) = 0$

$\Rightarrow x = -1$ or $\cancel{x = 3}$ $\Rightarrow f^{-1}(-2) = -1$

Also, $f'(x) = 2 - 2x$

$\Rightarrow \left.\frac{df^{-1}}{dx}\right|_{x=-2} = \frac{1}{2+2} = \frac{1}{4}$

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7. Find $\frac{d^{19}}{dx^{19}}(x \sin x)$

Note: $19 = 4(4) + 3$

$y = x \sin x$

$y' = \sin x + x \cos x$

$y'' = \cos x + \cos x - x \sin x = 2\cos x - x \sin x$

$y''' = -2\sin x - \sin x - x\cos x = -3\sin x - x\cos x$

$y^{(4)} = -3\cos x - \cos x + x \sin x = -4\cos x + x\sin x$

$\vdots$

$y^{(19)} = -19 \sin x - x \cos x$

8. If $f(x) = \tan^2\left(x^2 - \frac{3\pi}{4}\right)$, then find $f'(\sqrt{\pi})$

$f'(x) = 2\tan\left(x^2 - \frac{3\pi}{4}\right) * \sec^2\left(x^2 - \frac{3\pi}{4}\right) * 2x$

$\Rightarrow f'(\sqrt{\pi}) = 2\tan\frac{\pi}{4}\sec^2\frac{\pi}{4} * 2\sqrt{\pi}$

$= 8\sqrt{\pi}$