Math201.01, Quiz #3, Term 163

Name: Solutions

ID #: **Serial #:**

1. [5 points] Find the local maximum and minimum values and saddle points of

$$f(x,y) = x^3y + 12x^2 - 8y.$$

$\cdot \begin{cases} f_x = 0 \\ f_y = 0 \end{cases} \Rightarrow \begin{cases} 3x^2y + 24x = 0 \sim (1) \\ x^3 - 8 = 0 \sim (2) \end{cases}$ (1)

(1)

$(2) \Rightarrow (x-2)\underbrace{(x^2+2x+4)}_{\text{complex}} = 0$

$\Rightarrow x = 2$

$\overset{(1)}{\Rightarrow} 12y + (24)(2) = 0 \Rightarrow 12(y+4) = 0 \Rightarrow y = -4 \Rightarrow \boxed{(x,y) = (2,-4)}$

$\cdot\ f_{xx}(x,y) = 6xy + 24$; $f_{yy}(x,y) = 0$; $f_{xy}(x,y) = 3x^2$ (2)

$D(x,y) = f_{xx} \cdot f_{yy} - (f_{xy})^2 = 0 - (3x^2)^2 = -9x^4$

$D(2,-4) = -9(2)^4 = -144 < 0 \Rightarrow f$ has a Saddle point at $(2,-4)$

(0.5) (0.5)

$\cdot\ f(2,-4) = 8(-4) + 12(4) - 8(-4)$

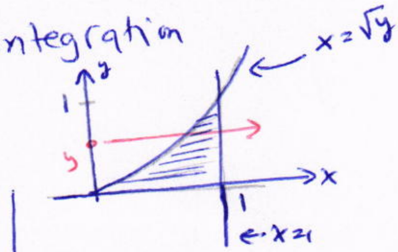
$= 4(-8+12+8)$

$= 48$

2. [4 points] Evaluate $\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3+1}\ dx\, dy$.

. Not easy to Integrate; Reverse the order of Integration

$R = \{(x,y) : \sqrt{y} \le x \le 1,\ 0 \le y \le 1\}$ Type II



Type I

$R: 0 \le x \le 1,\ 0 \le y \le x^2$

$\cdot \displaystyle\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3+1}\ dx\, dy = \int_0^1 \int_0^{x^2} \sqrt{x^3+1}\ dy\, dx$ (2)

$= \displaystyle\int_0^1 \sqrt{x^3+1} \cdot y \Big]_{y=0}^{y=x^2} dx$ (0.5)

$= \displaystyle\int_0^1 \sqrt{x^3+1} \cdot x^2\ dx$ (0.5)

$= \frac{1}{3} \cdot \frac{2}{3} (x^3+1)^{3/2} \Big]_0^1$ (0.5)

$= \frac{2}{9}\left(2^{3/2} - 1\right) = \frac{2}{9}(\sqrt{8} - 1) = \frac{2}{9}(2\sqrt{2} - 1)$

(0.5)

2

3. [6 points] Use Lagrange Multipliers to find the extreme values of $f(x,y) = x^2y$ subject to the constraint $x^2 + y^2 = 1$.

- $f(x,y) = x^2y$, $g(x,y) = x^2+y^2-1$
- Solve the System

$$\begin{cases} f_x = \lambda g_x \\ f_y = \lambda g_y \\ g = 0 \end{cases} \Longrightarrow \begin{cases} 2xy = \lambda(2x) & \text{—— (1)} \\ x^2 = \lambda(2y) & \text{—— (2)} \\ x^2+y^2-1 = 0 & \text{—— (3)} \end{cases}$$

1

0.5

- (1) $\Rightarrow 2xy - 2x\lambda = 0 \Rightarrow 2x(y-\lambda) = 0 \Rightarrow \boxed{x = 0 \text{ or } y = \lambda}$

* $x = 0 \overset{(3)}{\Longrightarrow} 0 + y^2 - 1 = 0 \Rightarrow y^2 = 1 \Rightarrow y = \pm 1 \Rightarrow \boxed{(x,y) = (0, \pm 1)}$ 1

(Note: the corresponding λ here is $\lambda = 0$ (from (2)))

** $y = \lambda \overset{(2)}{\Longrightarrow} x^2 = 2y^2 \overset{(3)}{\Longrightarrow} 2y^2+y^2-1 = 0 \Rightarrow y^2 = \frac{1}{3} \Rightarrow y = \pm\frac{1}{\sqrt{3}}$ 1

- $y = \frac{1}{\sqrt{3}} \Longrightarrow x^2 = \frac{2}{3} \Rightarrow x = \pm\sqrt{\frac{2}{3}} \Rightarrow \boxed{(x,y) = \left(\pm\sqrt{\frac{2}{3}}, \frac{1}{\sqrt{3}}\right)}$
- $y = -\frac{1}{\sqrt{3}} \Longrightarrow x^2 = \frac{2}{3} \Rightarrow x = \pm\sqrt{\frac{2}{3}} \Rightarrow \boxed{(x,y) = \left(\pm\sqrt{\frac{2}{3}}, -\frac{1}{\sqrt{3}}\right)}$ 1

0.5

- $f(0,1) = 0$, $f(0,-1) = 0$, $f\left(\sqrt{\frac{2}{3}}, \frac{1}{\sqrt{3}}\right) = \frac{2}{3\sqrt{3}}$, $f\left(-\sqrt{\frac{2}{3}}, \frac{1}{\sqrt{3}}\right) = \frac{2}{3\sqrt{3}}$

$f\left(\sqrt{\frac{2}{3}}, -\frac{1}{\sqrt{3}}\right) = -\frac{2}{3\sqrt{3}}$, $f\left(-\sqrt{\frac{2}{3}}, -\frac{1}{\sqrt{3}}\right) = -\frac{2}{3\sqrt{3}}$

0.5 , the max. value of f is $\frac{2}{3\sqrt{3}}$; It occurs at $\left(\pm\sqrt{\frac{2}{3}}, \frac{1}{\sqrt{3}}\right)$

0.5 The min ——— is $\frac{-2}{3\sqrt{3}}$; ——— $\left(\pm\sqrt{\frac{2}{3}}, \frac{-1}{\sqrt{3}}\right)$