

# KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS

## DEPARTMENT OF MATHEMATICS & STATISTICS

### AS 381 – Term 171

Date: October 25, 2017

Exam 1

Duration: 120 minutes

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Name:

ID #: \_ \_ \_ \_ \_

Section #: 01

Serial #: \_ \_

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#### Instructions:

- Write neatly and eligibly. You may lose points for messy work.
- Show all your work. No points for answers without justification.
- **Electronic approved calculators** are allowed. Mobiles are NOT allowed.
- Make sure that you have **7** pages of problems (Total of **5** Questions).

Question #	Score
1	/ 10
2	/ 10
3	/ 15
4	/ 20
5	/ 25
<b>Total</b>	<b>/ 80</b>

1. You are given the following survival function:

$$S_0(t) = \left(1 - \frac{t}{100}\right)^2, \quad 0 \leq t \leq 100$$

- (a) Find the probability that a new born infant survives to age 50.
- (b) Find the probability that a person aged 50 dies before attaining age 60.
- (c) Find the probability that a person aged now 50 survives to age 80 but dies before attaining age 90.

**(3 + 3 + 4 = 10 points)**

2. You are given:

(i)  $p_x = 0.90$

(ii)  $q_{x+3} = 0.15$

(iii)  ${}_2p_{x+3} = 0.68$

(iv)  $\mu_{x+t} = 0.12$ , where  $1 \leq t \leq 3$

Calculate the following:

(a)  $q_{x+4}$

(b)  ${}_5p_x$

(c)  ${}_{1.75}p_x$

**(3 + 4 + 3 = 10 points)**

3. Use the **Illustrative Life Table** with  $i = 6\%$ , to calculate the following:

(a)  ${}_{15|}q_{50}$

(b)  $A_{60:\overline{25}|}^1$

(c)  ${}^2A_{35:\overline{25}|}$

**(4 + 5 + 6 = 15 points)**

4. You are given:

(i) Mortality follows De Moivre's Law; i.e.  $l_x = \omega - x$ ,  $0 \leq x \leq \omega$  and the remaining future lifetime for  $x$  is  $T_x \sim U[0, \omega - x]$ .

(ii)  $\text{Var}(T_{60}) = 300$ .

(iii)  $Z_1$  is the present random variable for a whole life insurance on (50), with a death benefit of \$100 dollars payable at the moment of death.

(iv)  $Z_2 = \begin{cases} 100 v^{T_{30}}, & 0 \leq T_{30} \leq 15 \\ 200 v^{15}, & T_{30} > 15 \end{cases}$

(v) The constant force of interest is  $\delta = 0.05$ .

(a) Determine the implied limiting age  $\omega$ .

(b) Calculate the value of  $\dot{e}_{70}$ , the *complete* expectation of life at age 70.

(c) Find the mean and variance of  $Z_1$ .

(d) Describe in words (not formulas) the benefit associated with  $Z_2$ .

(e) Find the mean of  $Z_2$ .

**(4 + 3 + 6 + 3 + 4 = 20 points)**

5. You are given:

(i) The following excerpt of an incomplete life table:

$x$	60	61	62	63	64	65
$l_x$	100,000	99,900	$X$	99,500	99,250	99,000
$d_x$	100	200	200	$Y$	250	100

(ii)  $i = 0.04$

(iii)  $A_{66} = 0.77$

(iv) Deaths are uniformly distribution of between integer ages.

Calculate the following:

(a)  $X + Y$

(b)  ${}_{2.3}p_{61.2}$

(c)  $e_{60:\overline{5}|}$

(d)  $A_{60:\overline{5}|}$

(e)  $(IA)_{60:\overline{4}|}^1$

(f)  $A_{65}$

(g)  $A_{65}^{(12)}$

**(3 + 3 + 4 + 5 + 4 + 3 + 3 = 25 points)**