



King Fahd University of Petroleum & Minerals

**Second Major Examination**

<b>Faculty: Science</b>	<b>Department: Mathematics</b>
<b>Semester: 171</b>	<b>Course Name: Actuarial Risk &amp; Credibility Theory</b>
<b>Instructor: Abedalhay Elmughrabi</b>	<b>Course No: AS 483</b>
<b>Exam Date: December 11<sup>th</sup>, 2017</b>	<b>Exam Time: 07:00 PM – 09:00 PM</b>

<b>Student Name:</b>	<b>ID No.:</b>
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<b>Question No.</b>	<b>Question Full Marks</b>	<b>Question Obtained Marks</b>
<b>1</b>	<b>10 points</b>	
<b>2</b>	<b>10 points</b>	
<b>3</b>	<b>10 points</b>	
<b>4</b>	<b>10 points</b>	
<b>5</b>	<b>10 points</b>	
<b>6</b>	<b>10 points</b>	
<b>7</b>	<b>10 points</b>	
<b>8</b>	<b>10 points</b>	
<b>9</b>	<b>10 points</b>	
<b>10</b>	<b>10 points</b>	
<b>Total</b>	<b>100</b>	<b>Obtained Total:</b>



### **Exam Instructions**

1. Fill in all information required.
  2. The exam is composed of **10** questions.
  3. Only the following is allowed to be on your desk: SOA approved calculator, pen/pencil, eraser, and sharpener.
  4. Calculators cannot be exchanged during the examination.
  5. No use of smart devices with communications capabilities (mini laptops, pens, watches, phones, etc.)
  6. Cell phones must be turned off and placed under your bench facedown.
  7. No questions are allowed during the exam.
  8. All material related to the course should be put away
  9. Final correct answers have significant weights
  10. Answers without calculations/steps will receive zero marks.
  11. Be clean, neat and tidy, else your work may not be marked
  12. Students must not communicate with one another in any manner whatsoever during the examination.
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**GOOD LUCK**



**Questions 1 (10 Points):**

You are given

- (i) Losses follow an exponential distribution with the same mean in all years.
- (ii) The loss elimination ratio this year is 70%.
- (iii) The ordinary deductible for the next year is  $\frac{4}{3}$  of the current deductible.

Compute the LER for the next year?



**Questions 2 (10 Points):**

The graph of the density function for losses is

Calculate the LER for an ordinary deductible of 20?



**Questions 3 (10 Points):**

A compound Poisson distribution has  $\lambda=6$  and claims amount distribution as follows

x	P(x)
100	0.8
500	0.16
1000	0.04

Calculate the probability that aggregate claims will be exactly 500?



**Questions 4 (10 Points):**

$S_1$  has a compound Poisson distribution with parameter  $\lambda_1=1$  and discrete claim amount distribution  $P_1(x)$ :  $P_1(1)=0.75$  and  $P_1(5)=0.25$ .

$S_2$  has a compound Poisson distribution with parameter  $\lambda_2=1$  and discrete claim amount distribution  $P_2(x)$ :  $P_2(1)=0.5$  and  $P_2(5)=0.5$ .

$S_1$  and  $S_2$  are independent. Determine  $P(S_1+S_2 \leq 3)$



**Questions 5 (10 Points):**

For an aggregate loss distribution  $S$  :

(i) The number of claims has a negative binomial distribution with  $r = 16$  and  $\beta = 6$ :

(ii) The claim amounts are uniformly distributed on the interval  $(0, 8)$ :

(iii) The number of claims and claim amounts are mutually independent.

Using the normal approximation for aggregate losses, calculate the premium such that the probability that aggregate losses will exceed the premium is 5%.



**Questions 6 (10 Points):**

Let  $X_1, \dots, X_n$  be a random sample with a Poisson distribution with mean  $\theta$  and let  $\theta$  be Gamma  $(\alpha, \beta)$  with density

$$g(\theta) = \frac{\theta^{\alpha-1} e^{-\beta\theta} \beta^\alpha}{(\alpha-1)!} \quad \theta > 0$$

Find the Bayes estimator of  $\theta$  under the square error loss?



**Questions 7 (10 Points):**

You are given a sample of losses from an exponential distribution. However, if a loss is 1000 or greater, it is reported as 1000. The summarized sample is:

Reported Loss	Number	Total Amount
Less than 1000	62	28,140
1000	38	38,000
Total	100	66,140

Determine the maximum likelihood estimate of  $q$ , the mean of the exponential distribution.



**Questions 8 (10 Points):**

You are given:

- Losses follow a Burr distribution with  $\alpha = 2$ . A random sample of 15 losses is:  
195 255 270 280 350 360 365 380 415 450 490 550 575 590 615
- The parameters  $\gamma$  and  $\theta$  are estimated by percentile matching using the smoothed empirical estimates of the 30th and 65th percentiles.

Calculate the estimate of  $\gamma$ .



**Questions 9 (10 Points):**

For an insurance:

- (i) The number of losses per year has a Poisson distribution with  $\lambda = 10$ .
  - (ii) Loss amounts are uniformly distributed on  $(0, 10)$
  - (iii) Loss amounts and the number of losses are mutually independent.
  - (iv) There is an ordinary deductible of 4 per loss.
- Calculate the variance of aggregate payments in a year.



**Questions 10 (10 Points):**

The number of claims in a period has a geometric distribution with mean 3. The amount of each claim  $X$  follows  $\Pr(X = x) = 0.25; x = 1, 2, 3, 4$ : The number of claims and the claim amounts are independent.  $S$  is the aggregate claim amount in the period.

Calculate  $F_S(3)$ :