King Fahd University of Petroleum & Minerals

Final Examination

Faculty: Science	**Department: Mathematics**
Semester: 171	**Course Name: Actuarial Risk & Credibility Theory**
Instructor: Abedalhay Elmughrabi	**Course No: AS 483**
Exam Date: January 7th, 2017	**Exam Time: 08:00 AM – 11:00 AM**

Student Name:	**ID No.:**

Question No.	Question Full Marks	Question Obtained Marks	Question No.	Question Full Marks	Question Obtained Marks
1	8 points		11	8 points	
2	9 points		12	8 points	
3	9 points		13	8 points	
4	8 points		14	8 points	
5	8 points		15	8 points	
6	8 points		16	8 points	
7	9 points		17	8 points	
8	8 points				
9	8 points				
10	9 points				

Obtained Total:

/ 140

Exam Instructions

1. Fill in all information required.
2. The exam is composed of **17** questions.
3. Only the following is allowed to be on your desk: SOA approved calculator, pen/pencil, eraser, and sharpener.
4. Calculators cannot be exchanged during the examination.
5. No use of smart devices with communications capabilities (mini laptops, pens, watches, phones, etc.)
6. Cell phones must be turned off and placed under your bench facedown.
7. No questions are allowed during the exam.
8. All material related to the course should be put away
9. Final correct answers have significant weights
10. Answers without calculations/steps will receive zero marks.
11. Be clean, neat and tidy, else your work may not be marked
12. Students must not communicate with one another in any manner whatsoever during the examination.

GOOD LUCK

Questions 1 (8 Points):

Let S be a compound Poisson distribution with parameter λ= 0.04 and individual claim distribution given by

x	$f_X(x)$
1	0.5
2	0.4
3	0.1

Show that:

$$f_S(n) = \frac{1}{n}[0.02 f_S(n-1) + 0.032\, f_S(n-2) + 0.012\, f_S(n-3)]$$

Questions 2 (9 Points):

For a portfolio of insurance risks, aggregate losses per year per exposure follow a normal distribution with mean θ and standard deviation 1000, with θ varying by class as follows:

Class	θ	Percent of Risks in Class
X	2000	60%
Y	3000	30%
Z	4000	10%

A randomly selected risk has the following experience over three years:

Year	Number of Exposures	Percent of Risks in Class
1	24	24,000
2	30	36,000
3	26	28,000

Calculate the Bühlmann-Straub estimate of the mean aggregate losses per year per exposure in Year 4 for this risk.

Questions 3 (9 Points):

The partial credibility approach is applied to a data of 50 claim amounts. It is assumed that the claim amount distribution is uniform on the interval $[0,\theta]$. The full credibility standard is to be within 5% of the expected claim amount 90% of the time. The partial credibility factor Z is found. After 25 additional claim amounts are recorded, the claim amount distribution is revised to be uniform on the interval $[0,1.2\theta]$. The revised partial credibility factor Z^* is found. Fin the ratio $\frac{Z}{Z^*}$

Questions 4 (8 Points):

You are given the following probability density function:

$$f(x) = \frac{x^2}{9} 1_{[0 \le x \le 3]}$$

You are to simulate three observations from the distribution using the inversion method and the following three random number from the uniform distribution on (0,1): 0.008, 0.729, 0.125. Using the three simulated observations, estimate the mean of the distribution.

Questions 5 (8 Points):

A group dental policy has a negative binomial claim count distribution with mean 300 and variance 800.
Ground-up severity is given by the following table:

Severity	Probability
40	0.25
80	0.25
120	0.25
200	0.25

You expect severity to increase 50% with no change in frequency. You decide to impose a per claim deductible of 100.

Calculate the expected total claim payment after these changes.

Questions 6 (8 Points):

The loss random variable X has an exponential distribution with mean $\frac{1}{\lambda}$ and an ordinary deductible is applied to all losses. The variance of the cost per payment random variable (excess loss random variable) is 25,600. The variance of the cost per loss random variable is 20,480. Find the amount of the deductible d?

Questions 7 (9 Points):

You are given total claims for two policyholders:

	Year			
Policyholder	1	2	3	4
X	730	800	650	700
Y	655	650	625	750

Using the nonparametric empirical Bayes method, determine the Bühlmann credibility premium for Policyholder Y.

(A) 655
(B) 670
(C) 687
(D) 703
(E) 719

Questions 8 (8 Points):

Losses have a lognormal distribution with $\mu=7$ and $\sigma=2$. There is a deductible of 2,000, and 10 losses are expected each year. Determine the loss elimination ratio. If there is uniform inflation of 20% but the deductible remains at 2000, how many payments will be expected?

Questions 9 (8 Points):

Let $X_1, X_2, X_3 \ldots X_n$ be a random sample from Normal (θ, σ^2) and let θ be normal (μ, c^2).

Find the Bayes estimate of θ under the square error loss?

Questions 10 (9 Points):

An insurance company is revising rates based on old data. The expected number of claims for full credibility is selected so that the observed total claims will be within 5% of the true value 90% of the time. Individual claim amounts have pdf 1/200,000, 0<x<200,000, and the number of claims has the poison distribution. The recent experience consists of 1,082 claims. Determine the credibility, Z, to be assigned to the recent experience. Use the normal approximation.

Questions 11 (8 Points):

An insurer writes a large book of home warranty policies. You are given the following information regarding claims filed by insureds against these policies:

(i) A maximum of one claim may be filed per year.
(ii) The probability of a claim varies by insured, and the claims experience for each insured is independent of every other insured.
(iii) The probability of a claim for each insured remains constant over time.

(iv) The overall probability of a claim being filed by a randomly selected insured in a year is 0.10.

(v) The variance of the individual insured claim probabilities is 0.01.

An insured selected at random is found to have filed 0 claims over the past 10 years.

Determine the Bühlmann credibility estimate for the expected number of claims the selected insured will file over the next 5 years.

Questions 12 (8 Points):

X is a discrete random variable with probability function which is a member of the (a,b,0) class of distributions. You are given:

i. P(X=0)=P(X=1)=0.25
ii. P(X=2)=0.1875

Calculate P(X=3)

Questions 13 (8 Points):

You are planning a simulation to estimate the mean of a nonnegative random variable. It is known that the population standard deviation is 20% larger than the population mean. Use the Central limit theorem to estimate the smallest number of trials needed so that you will be at least 95% confident that the simulated value mean is within 5% of the population mean?

Questions 14 (8 Points):

A dental plan is designed so that a deductible of 100 is applied to annual dental charges. The reimbursement to the insured is 80% of the remaining dental charges subject to an annual maximum reimbursement of 1000.

You are given

i. The annual dental charges for each insured are exponentially distributed with mean 100.

ii. Use the following Uniform (0,1) random numbers and the inversion method to generate four values of annual dental charges.

0.3 0.92 0.7 0.08

Calculate the average annual reimbursement for this simulation?

Questions 15 (8 Points):

You are given:

(i) The probability that an insured will have at least one loss during any year is p.
(ii) The prior distribution for p is uniform on [0,0.5].
(iii) An insured is observed for 8 years and has at least one loss every year.

Determine the posterior probability that the insured will have at least one loss during Year 9.

Questions 16 (8 Points):

You are given:

(i) Two risks have the following severity distributions:

Amount of Claim	Probability of Claim Amount for Risk 1	Probability of Claim Amount for Risk 2
250	0.5	0.7
2,500	0.3	0.2
60,000	0.2	0.1

(ii) Risk 1 is twice as likely to be observed as Risk 2.
A claim of 250 is observed.

Determine the Bühlmann credibility estimate of the second claim amount from the same risk.

Questions 17 (8 Points):

The distribution of a loss, X, is a 2-point mixture:

(i) With probability 0.6, X_1 is a Pareto distribution with parameters $\alpha = 3$ and $\theta = 900$
(ii) With probability 0.4, X_2 is a Pareto distribution with parameters $\alpha = 5$ and $\theta = 1500$

Determine Pr (X > 1000)