(1) (5 points) Find all vertical and horizontal asymptotes of the function \( f(x) = \frac{x}{x+1} - \frac{1}{x(x+1)} = \frac{x^2 - 1}{x(x+1)} = \frac{(x-1)(x+1)}{x(x+1)} \)

Vertical asymptotes: Check at \( x = 0 \) and \( x = -1 \)

\[
\lim_{x \to 0^+} \frac{(x-1)(x+1)}{x(x+1)} = -\infty \quad \Rightarrow \quad x = 0 \text{ is VA}
\]

\[
\lim_{x \to -1^-} \frac{(x-1)(x+1)}{x(x+1)} = \frac{-2}{-1} = 2 \quad \Rightarrow \quad x = -1 \text{ is NOT VI}
\]

Horizontal Asymptote: \( \lim_{x \to \pm \infty} \frac{x^2 - 1}{x(x+1)} = \lim_{x \to \pm \infty} \frac{x - \frac{1}{x}}{x + 1} = 1 \quad \Rightarrow \quad y = 1 \text{ is HA} \)

(2) (5 points) If \( f'(3) = 5 \) then \( \lim_{x \to 3} \frac{f(x) - f(3)}{x^2 - 9} = \)

\[
= \lim_{x \to 3} \frac{f(x) - f(3)}{x - 3} \cdot \frac{1}{x + 3}
\]

\[
= \lim_{x \to 3} \frac{f(x) - f(3)}{x - 3} \cdot \lim_{x \to 3} \frac{1}{x + 3}
\]

\[
= f'(3) \cdot \frac{1}{6} = \frac{5}{6}
\]

(3) (5 points) Use the Intermediate Value Theorem to show that the two functions \( f(x) = x^2 - 1 \) and \( g(x) = \sqrt{x} + 2 \)

Intersect between 0 and 2.

For the two functions to intersect, \( f(x) = g(x) \) or \( x^2 - 1 = \sqrt{x} + 2 \)

Let \( H(x) = x^2 - 1 - \sqrt{x} + 2 \)

(i) \( H(x) \) is continuous on \([0, 2]\)

(ii) \( H(0) = 0 - 1 - \sqrt{2} = -1 - \sqrt{2} < 0 \)

(iii) \( H(2) = 2^2 - 1 - \sqrt{2 + 2} = 3 - 2\sqrt{2} > 0 \)

\[ \therefore \text{By the IVT, there is a number } c \text{ between 0 and 2 such that } H(c) = 0 = c^2 - 1 - \sqrt{c + 2} \]

\[ \text{OR } c^2 - 1 = \sqrt{c + 2} \]
(4) (5 points) Evaluate the limit: \( \lim_{x \to \infty} \left[ \sqrt{x^2 + 4x} - x \right] = \)

\[
= \lim_{x \to \infty} \left[ \frac{\sqrt{x^2 + 4x} - x}{\sqrt{x^2 + 4x} + x} \right] \cdot \frac{\sqrt{x^2 + 4x} + x}{\sqrt{x^2 + 4x} + x} \cdot \frac{\frac{\sqrt{x^2 + 4x}}{x} - \frac{1}{x}}{\frac{\sqrt{1 + \frac{4}{x}}}{x} + \frac{1}{x}}
\]

\[
= \lim_{x \to \infty} \left[ \frac{4}{\left(1 + \frac{4}{x}\right)^{\frac{1}{2}} + 1} \right]
\]

\[
= \lim_{x \to \infty} \frac{4}{\sqrt{1 + \frac{4}{x}} + 1} = \frac{4}{1 + 1} = 2
\]

(5) (7 points) Find all points of discontinuity of the function, if any exists, and state the type of discontinuity of each one.

\[
f(x) = \begin{cases} \frac{1}{x + 1} & \text{if } x < -1 \\ \sin(x^2 - 1) & \text{if } -1 \leq x \leq 1 \\ \sqrt{x - 1} & \text{if } 1 < x \end{cases}
\]

Note that \( f \) is continuous on \((-\infty, -1) \cup (-1, 1) \cup (1, \infty)\)

At \( x = -1 \):

\[
\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} \frac{1}{x + 1} = -\infty
\]

\[
\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} \sin(x^2 - 1) = 0
\]

\[\therefore \text{ at } x = -1 \text{ we have infinite discontinuity} \]

At \( x = 1 \):

\[
\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \frac{1}{x + 1} \sin(x^2 - 1) = 0
\]

\[
\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \sqrt{x - 1} = 0
\]

\[\therefore \lim_{x \to 1} f(x) = 0 = f(1). \text{ Thus } f(x) \text{ is continuous at } 1.\]
(6) (5 points) Let \( f(x) = \sqrt{2x + 1} \). Use the definition of the derivative to find \( f'(4) \).

\[
f'(4) = \lim_{x \to 4} \frac{f(x) - f(4)}{x - 4} = \lim_{x \to 4} \frac{\sqrt{2x + 1} - 3}{x - 4}
\]

\[
= \lim_{x \to 4} \frac{\sqrt{2x + 1} - 3}{x - 4} \cdot \frac{\sqrt{2x + 1} + 3}{\sqrt{2x + 1} + 3}
\]

\[
= \lim_{x \to 4} \frac{2x + 1 - 9}{(x - 4)(\sqrt{2x + 1} + 3)}
\]

\[
= \lim_{x \to 4} \frac{2(x - 4)}{(x - 4)(\sqrt{2x + 1} + 3)} = \frac{2}{\sqrt{8 + 1} + 3}
\]

\[
= \frac{2}{3 + 3} = \frac{1}{3}
\]

(7) (6 points) Determine if the function \( f(x) = \left\lceil \frac{x}{1 - x} \right\rceil \) is continuous at \( x = -1 \).

\[
f'(-1) = \left\lceil \frac{-1}{1 - (-1)} \right\rceil = \left\lceil \frac{-1}{2} \right\rceil = -1
\]

Now: \( \lim_{x \to -1^-} \left\lceil \frac{x}{1 - x} \right\rceil = \left\lceil \lim_{x \to -1^-} \frac{x}{1 - x} \right\rceil = \left\lceil \frac{1}{2} \right\rceil = -1
\]

\[
\lim_{x \to -1^+} \left\lceil \frac{x}{1 - x} \right\rceil = \left\lceil \lim_{x \to -1^+} \frac{x}{1 - x} \right\rceil = \left\lceil \frac{1}{2} \right\rceil = -1
\]

\[
\therefore \lim_{x \to -1} f(x) = -1 = f(-1)
\]

So \( f(x) \) is continuous at \( x = -1 \).
(8) (5 points) Find the rate of change in the surface area of a sphere with respect to the radius when the radius is 5. Here the surface area of a sphere is \( S(r) = 4\pi r^2 \).

\[
\frac{dS}{dr}\bigg|_{r=5} = S'(5) = \lim_{r \to 5} \frac{S(r) - S(5)}{r - 5}
\]

\[
= \lim_{r \to 5} \frac{4\pi r^2 - 4\pi (25)}{r - 5}
\]

\[
= 4\pi \lim_{r \to 5} \frac{r^2 - 25}{r - 5}
\]

\[
= 4\pi \lim_{r \to 5} \frac{(r-5)(r+5)}{r-5}
\]

\[
= 4\pi (5+5)
\]

\[
= 40\pi
\]

(9) (7 points) A particle is moving along a horizontal line with position function \( s(t) = t - t^2 \), for \( t \geq 0 \) where \( s(t) \) is measured in meters and \( t \) in seconds. Find the distance traveled during the first two seconds. Also find the speed at \( t = 2 \).

\[
V(t) = s'(t) = \lim_{h \to 0} \frac{s(t+h) - s(t)}{h} = \lim_{h \to 0} \frac{(t+h) - (t) - (t+t^2) - (t-t^2)}{h}
\]

\[
= \lim_{h \to 0} \frac{t + h - t - 2t - t^2}{h}
\]

\[
= \lim_{h \to 0} \frac{h(1 - 2t + h)}{h}
\]

\[
= 1 - 2t
\]

\[
V(t) = 1 - 2t = 0 \Rightarrow t = \frac{1}{2}
\]

Thus: \( V(t) =, \frac{1}{2} \)

The particle reverses direction at \( t = \frac{1}{2} \).

Distance traveled = \( |S(\frac{1}{2}) - S(0)| + |S(2) - S(\frac{1}{2})| \)

\[
= \left| \frac{1}{2} - \frac{1}{4} \right| + \left| (2) - (\frac{1}{2}) - (\frac{1}{2}) \right|
\]

\[
= \frac{1}{4} + 2 \frac{1}{4} = \frac{5}{2} \text{ m}
\]

\[
\text{Speed } |V(2)| = 1 - 2(2) = 1 - 3 = 3 \text{ m/s}
\]