Solution:

The slope of the line tangent to the parabola at \((-1,0)\) is \(y'(-1) = -2x\big|_{x=-1} = 2\).

Thus, the slope of the normal line is \(m = -\frac{1}{2}\).

The equation of the normal line is

\[ y - 0 = -\frac{1}{2} (x + 1) \]

Now, the parabola and the line intersect when

\[ 1 - x^2 = y = -\frac{1}{2} (x + 1) \]

That is, when \(2x^2 - x - 3 = 0\)

\( (x+1)(2x-3) = 0 \).

Then \(x = -1\) and \(x = \frac{3}{2}\).

\(x = -1\) gives the first point \((-1,0)\).
\(x = \frac{3}{2}\) gives the second point \(\left(\frac{3}{2}, -\frac{5}{4}\right)\).
(2) (7 points) Let \( g(x) = \frac{x + e^x f(x)}{\sqrt{2x} + 1} \) and \( g'(0) = 2 \). Find \( f'(0) \).

\[
\frac{g'(x)}{2x + 1} = \frac{1 + e^x f(x) + e^x f'(x)}{2\sqrt{2x} + 1} - \left[ x + e^x f(x) \right] \frac{2}{2\sqrt{2x} + 1}
\]

\[
9 = g'(0) = 1 + f(0) + f'(0) - f(0)
\]

\[
= 1 + f'(0)
\]

\[
\Rightarrow f'(0) = 2 - 1 = 1
\]

(3) (7 points) Find the limit if it exists: \( \lim_{x \to 2} \frac{\cos(x - 2) \sin(x - 2)}{x^2 + x - 6} \)

\[
\lim_{x \to 2} \frac{\cos(x - 2) \sin(x - 2)}{x^2 + x - 6} = \lim_{x \to 2} \frac{\cos(x - 2) \sin(x - 2)}{(x + 3)(x - 2)}
\]

\[
= \lim_{x \to 2} \frac{\cos(x - 2)}{x + 3} \cdot \lim_{x \to 2} \frac{\sin(x - 2)}{x - 2}
\]

\[
= \frac{\cos 0}{2 + 3} \cdot \lim_{\theta \to 0} \frac{\sin \theta}{\theta}
\]

\[
= \frac{1}{5} \cdot (1) = \frac{1}{5}
\]
(4) (7 points) If \( f(x) = \sin x \cos x \), find \( f^{(52)}(x) \).

**Solution:**

\[
f^{(52)}(x) = \frac{1}{2} \sin 2x
\]

**Note that 52 is divisible by 4**

\[
f^{(51)}(x) = 2 \sin 2x
\]

(5) (7 points) If \( y = \sin^3(\csc^5 x) \), find \( \frac{dy}{dx} \).

\[
y' = 3 \sin^2(\csc^5 x) \cdot \cos(\csc^5 x) \left[-\csc^5 x \cot x^5 \cdot 5x^4\right]
\]
(6) (7 points) Find the equation of the line tangent to the curve 
\[ x^3 + \tan^{-1}(xy) = x^2 + y^2 \]
at the point (1,0).

**Solution:** differentiate both sides implicitly:
\[
3x^2 + \frac{1}{1+(xy)^2} \cdot [xy' + y] = 2x + 2yy'
\]

Substitute \( x = 1, \ y = 0 \):

\[
3 + \frac{1}{1+0} \left[ y' + 0 \right] = 2 + 0 \cdot y' \]

\[
3 + y' = 2 \quad \left\{ \text{Equation of the tangent} \right\}
\]

\[
y' = -1 \quad \left\{ \begin{array}{l}
\text{i} \quad y - 0 = -1(x - 1) \\
\text{or} \quad y = -x + 1
\end{array} \right.
\]

(7) (7 points) If \( y = (x + \sec x)^{(1+x)} \), find \( y'(0) \).

**Solution:** when \( x = 0 \Rightarrow y = (0 + 1)^1 = 1 \)

Take the ln of both sides:

\[
\ln y = \ln (x + \sec x)
\]

\[
= (1 + x) \ln (x + \sec x)
\]

Differentiate both sides implicitly:

\[
\frac{y'}{y} = \ln(x + \sec x) + \frac{1 + x}{x + \sec x} \cdot [1 + \sec x \cdot \tan x]
\]

Substitute \( x = 0, \ y = 1 \):

\[
y' = \ln(0+1) + \frac{1+0}{0+1} \cdot [1 + 1(0)]
\]

\[
y' = (\ln 1) + 1 = 0 + 1 = 1
\]