Serial No.:_______ Student Name: ______________________________ Student Number: ________

Instructor: M. Z. Abu-Sbeih Math 101- Q3 Date: 14-11-2017

SHOW ALL YOUR WORK. NO CREDITS FOR ANSWERES WITHOUT JUSTIFICATIONS

(1) (8 points) Where does the normal line to the parabola $y = 1 - x^2$ at the point (-1,0) intersect the parabola a second time?

Solution:

The slope of the line tangent to the parabola at (-1, 0) is $y'(-1) = -2x\big|_{x=-1} = 2$.

Thus, the slope of the normal line is $m = -\frac{1}{2}$.

The equation of the normal line is

$$y - 0 = -\frac{1}{2}(x+1)$$

Now, the parabola and the line intersect when

$$1 - x^2 = y = -\frac{1}{2}(x+1)$$

That is, when $2x^2 - x - 3 = 0$

$$(x+1)(2x-3) = 0.$$

Then $x = -1$ and $x = \frac{3}{2}$

$x = -1$ gives the first point $(-1, 0)$.

$x = \frac{3}{2}$ gives the second point $\left(\frac{3}{2}, -\frac{5}{4}\right)$.

(2) (7 points) Let $g(x) = \frac{x+e^x f(x)}{\sqrt{2x+1}}$ and $g'(0) = 2$. Find $f'(0)$.

Solution:

$$g'(x) = \frac{\sqrt{2x+1}\left[1 + e^x f(x) + e^x f'(x)\right] - \left[x + e^x f(x)\right] \cdot \frac{2}{2\sqrt{2x+1}}}{2x+1}$$

$$2 = g'(0) = \sqrt{0+1}\left[1 + e^0 f(0) + e^0 f'(0)\right] - \left[0 + e^0 f(0)\right] \frac{1}{\sqrt{0+1}}$$

$$= 1 + f(0) + f'(0) - f(0)$$

$$= 1 + f'(0)$$

$$\Rightarrow f'(0) = 2 - 1 = 1$$

(3) (7 points) Find the limit f it exists: $\lim_{x \to 2} \frac{\cos(x-2)\sin(x-2)}{x^2+x-6}$

Solution:

$$\lim_{x \to 2} \frac{\cos(x-2)\sin(x-2)}{x^2+x-6} = \lim_{x \to 2} \frac{\cos(x-2)\sin(x-2)}{(x+3)\ (x-2)}$$

$$= \lim_{x \to 2} \frac{\cos(x-2)}{x+3} \cdot \lim_{x \to 2} \frac{\sin(x-2)}{x-2}.$$

put $\theta = x-2$ as $x \to 2$, $\theta \to 0$.

$$= \frac{\cos 0}{2+3} \cdot \lim_{\theta \to 0} \frac{\sin\theta}{\theta}$$

$$= \frac{1}{5} \cdot (1) = \frac{1}{5}$$

(4) (7 points) If $f(x) = \sin x \cos x$, find $f^{(52)}(x)$.

Solution:

$$f(x) = \frac{1}{2} \sin 2x$$

$$f^{(52)}(x) = \frac{1}{2} \, 2^{52} \sin(2x)$$

$$= 2^{51} \sin 2x$$

(Note that 52 is divisible by 4)

(5) (7 points) If $y = sin^3(\csc x^5)$, find $\frac{dy}{dx}$.

$$y' = \left[3 \sin^2 (\csc x^5)\right] \cdot \left[\cos (\csc x^5)\right]\left[-\csc x^5 \cot x^5\right] \cdot \left[5x^4\right]$$

(6) (7 points) Find the equation of the line tangent to the curve

$$x^3 + tan^{-1}(xy) = x^2 + y^2$$

at the point (1,0).

Solution: differentiate both sides implicitly:

$$3x^2 + \frac{1}{1+(xy)^2} \cdot [xy' + y] = 2x + 2yy'$$

Substitute $x = 1$, $y = 0$:

$$3 + \frac{1}{1+0} [y' + 0] = 2 + 0 \cdot y'$$

$$3 + y' = 2$$

$$y' = -1$$

Equation of the tangent is $y - 0 = -1(x-1)$ or $y = -x+1$

(7) (7 points) If $y = (x + \sec x)^{(1+x)}$, find $y'(0)$.

Solution: When $x=0 \Rightarrow y = (0+1)^1 = 1$

Take the ln of both sides:

$$\ln y = \ln (x + \sec x)^{(1+x)}$$

$$= (1+x) \ln (x + \sec x)$$

Differentiate both sides implicitly:

$$\frac{y'}{y} = \ln(x+\sec x) + \frac{1+x}{x + \sec x} \cdot [1 + \sec x \tan x]$$

Substitute $x = 0$, $y = 1$

$$y' = \ln(0+1) + \frac{1+0}{0+1} \cdot [1 + 1(0)]$$

$$= (\ln 1) + 1 = 0 + 1 = 1$$