Serial No.:______ Student Name:______________________ Student Number: ______

Instructor: M. Z. Abu-Sbeih Math 101- Q4 Date: 19-12-2017

**SHOW ALL YOUR WORK. NO CREDITS FOR ANSWERES WITHOUT JUSTIFICATIONS**

**Problem 1** (10 points) Find the limit if it exists. Write $\infty$ or $(-\infty)$ when appropriate:

a. $\lim\limits_{x \to 0^+} \sqrt{x}\,(1+\ln x)$

$$\overset{0\cdot\infty}{=} \lim_{x\to 0^+} \frac{1+\ln x}{x^{-1/2}}$$

$$\underset{L'H}{\overset{\frac{\infty}{\infty}}{=}} \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{1}{2}x^{-3/2}}$$

$$= \lim_{x\to 0^+} -2\frac{x^{3/2}}{x}$$

$$= \lim_{x\to 0^+} -2\sqrt{x} = 0$$

b. $\lim\limits_{x \to 0^+} \left(\dfrac{\ln x}{x}\right)^{\frac{1}{\ln x}}$

Let $y = \left(\dfrac{\ln x}{x}\right)^{\frac{1}{\ln x}}$

$$\ln y = \frac{1}{\ln x}\ln\left(\frac{\ln x}{x}\right)$$

$$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \frac{\ln\left(\frac{\ln x}{x}\right)}{\ln x}$$

$$\overset{\frac{\infty}{\infty}}{=} \lim_{x\to 0^+} \frac{\ln(\ln x) - \ln x}{\ln x}$$

$$= \lim_{x\to 0^+} \frac{\frac{1}{x\ln x} - \frac{1}{x}}{\frac{1}{x}}$$

$$= \lim_{x\to 0^+} \left(\frac{1}{\ln x} - 1\right) = 0 - 1 = -1$$

$$\therefore \lim_{x\to 0^+} y = e^{-1} = \frac{1}{e}$$