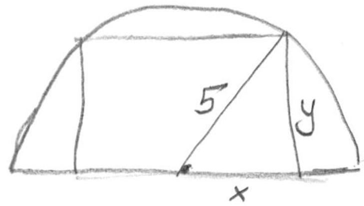
Serial No.:______ Student Name:________________________ Student Number: ______
Instructor: M. Z. Abu-Sbeih Math 101- Q5 Date: 25-12-2017

SHOW ALL YOUR WORK. NO CREDITS FOR ANSWERES WITHOUT JUSTIFICATIONS

Problem 1 (15 points) A rectangle is constructed with its base on the diameter of a semicircle with radius 5 and its two other vertices on the semicircle. What are the dimensions of the rectangle with maximum area?

$A = 2xy = 2x\sqrt{25-x^2}$; $0 \le x \le 5$



$A' = 2\sqrt{25-x^2} + 2x \dfrac{-2x}{2\sqrt{25-x^2}}$

$= \dfrac{2[25-2x^2]}{\sqrt{25-x^2}}$

$y = \sqrt{25-x^2}$

$x = \frac{5}{\sqrt{2}}$, $y = \sqrt{25 - \frac{25}{2}} = \frac{5}{\sqrt{2}}$: $A(\frac{5}{\sqrt{2}}) = 25$, $A(0) = 0 = A(5)$

$\therefore$ Max area when dimensions $\frac{10}{\sqrt{2}} \times \frac{5}{\sqrt{2}}$

Question 2: (35points) Consider the function

$$y = f(x) = \frac{x^3}{x^2-1} \quad \text{with} \quad f'(x) = \frac{x^2(x^2-3)}{(x^2-1)^2} \quad \text{and} \quad f''(x) = \frac{2x(x^2+3)}{(x^2-1)^3}$$

a. (3 Points) Find the intercepts.

$x = 0$, $y = 0$

b. (5 Points) Find the asymptotes if any exist.

Horizontal:

None because: $\lim\limits_{x \to \mp\infty} \frac{x^3}{x^2-1} = \mp\infty$

Vertical:

$x = 1$ because $\lim\limits_{x \to 1^+} \frac{x^3}{x^2-1} = \infty$

$x = -1$ " $\lim\limits_{x \to -1^+} \frac{x^3}{x^2-1} = \infty$

Slant:

None.

c. (2 Points) Find the critical numbers.

$x = 0$ $x = \mp\sqrt{3}$

$(1.73, 2.6)$

$(0,0)$, $(\sqrt{3}, \frac{3\sqrt{3}}{2})$, $(-\sqrt{3}, -\frac{3\sqrt{3}}{2})$

d. (4 Points) Find intervals where the function is increasing and those where it is decreasing.

f': + $-\sqrt{3}$ − -1 − 0 − 1 − $\sqrt{3}$ +

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e. (2 Points) Find the local maximum and local minimum of the function.

at $x = -\sqrt{3}$ L. max

$x = \sqrt{3}$ L min

at $x = 0$ None.

f. (5 Points) Discuss the concavity of the function and find the infection points if any exist.

Inflection points: (0,0).

$x = 0 \rightarrow y = 0$

i.e (0,0).

f'': $-$ $+$ $-$ $+$ (sign changes at -1, 0, 1)

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g. (10 Points) Sketch the graph of the function. Clearly indicate the **critical numbers**, **extrema** and **inflection points** on the graph.