Name:

KFUPM ID:

Exercise 1

Let m and n be two natural numbers. Find (if it exists)

$$\lim_{x\to 0}\frac{\sqrt{1+x^m}-\sqrt{1-x^m}}{x^n},$$

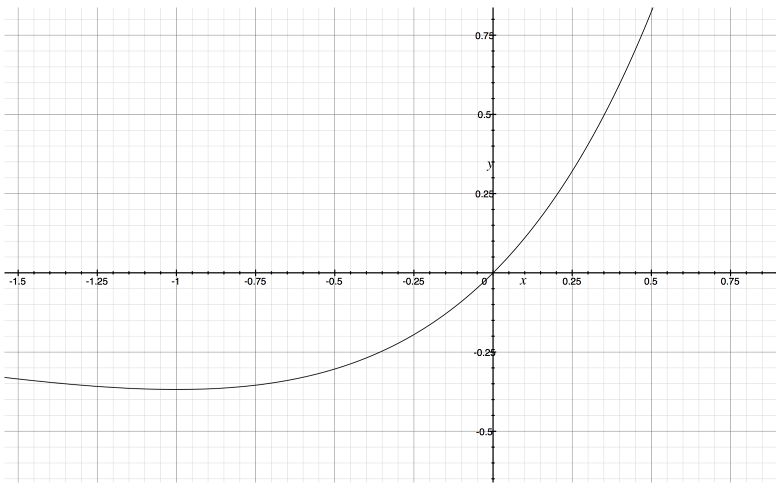
in the following cases

- $m > n$
- $m = n$
- $m < n$

Exercise 2

Let $f(x) = x\,e^x$

1. Determine graphically, the largest $\delta > 0$ so that $\left|x+\frac{1}{2}\right| < \delta \Longrightarrow |f(x) - \lim_{x\to-\frac{1}{2}} f(x)| < 0.1$



Graph of $y = xe^x$

Notice that $e \simeq 2,718$

2. Use the fact that for all $x > 0$, we have $e^x > \frac{x^2}{2}$ to find

$$\lim_{x\to+\infty}\frac{e^x}{x} = +\infty.$$

3. Deduce

$$\lim_{x\to-\infty} xe^x.$$

4. Determine (if any) the vertical and horizontal asymptotes of f ?
5. Show that for all $a > 0$, there exists at least one point $A(x_A, f(x_A))$ satisfying $0 < x_A < a$, such that the tangent to the curve $y = f(x)$ at A is parallel to the line $y = e^a(x - \ln a)$.

   Hint= Use the intermediate value Theorem

Exercise 3

Let $f(x) = e^x + x^3 \cos(2x\pi)$. Show that the equation

$$f(x) = \frac{e+1}{2},$$

has at least one root in $(0,1)$.

We recall that $e = 2.718....$

Exercise 4

We introduce the following function

$$f(x) = \begin{cases} e^x & \text{if } x < 0, \\ \\ ax^2 + bx + c & \text{otherwise.} \end{cases}$$

1. Determine **if there exist** a, b and c so that f is twice differentiable with continuous derivatives.
2. Show that $f^{(3)}$ (the third derivative of f) is not continuous at 0.

Hint: f'' should be continuous on $\mathbb{R}$ which, in particular, means that f' should be differentiable (thus continuous) on $\mathbb{R}$ which also means that f should be differentiable (thus continuous) on $\mathbb{R}$.

Exercise 5

Let f be a differentiable function such that $f(x) \neq \pm x$. We define

$$h(x) = \frac{\sqrt{x}}{f(x)+x}, \qquad \text{and} \qquad g(x) = \frac{x^2}{f(x)-x}.$$

- Find the expressions for the derivatives of h and g.

Exercise 6 [Optional]

Find

$$\lim_{x\to 0} \frac{\sin(3x)\sin(7x)\cos(x-\frac{\pi}{4})}{x\sin(\pi x)}.$$